(1) Exercises from Demo 2 (except 2a,2b).

(2) **explosions**

Let $(S_n)_{n=1}^{\infty}$ be a sequence of independent random variables where each S_n is exponentially distributed with parameter $\lambda_n > 0$. We define a stochastic process $(X_t)_{t\geq 0}$ as follows:

$$\begin{aligned} T_0 &:= 0, \\ T_n &:= S_1 + ... + S_n, \quad n \in \mathbb{N}, \\ X_0 &:= 0, \\ X_t &:= \sum_{n=1}^{\infty} n \mathbb{1}_{[T_n, T_{n+1})}(t), \quad t > 0. \end{aligned}$$

We say that the path $t \mapsto X_t(\omega)$ *explodes*, if $\tau(\omega) := \sum_{n=1}^{\infty} S_n(\omega) < \infty$. Explain (use exercise (3) from Demo 2)

(a) under which conditions and why the process X either a.s. explodes or a.s. does not explode,

(b) why a process for which it holds $\lambda_n = \lambda$ for all $n \in \mathbb{N}$ (which is, in fact the Poisson process) does a.s. not explode.