(2a) We have that

$$\text{explosion} = \left\{ \omega \in \Omega : \sum_{n=1}^{\infty} S_n(\omega) < \infty \right\}$$

$$= \left\{ \omega \in \Omega \ \sum_{n=N}^{\infty} S_n(\omega) < \infty \right\}$$

for all $N = 1, 2, \dots$ Hence

$$\text{explosion} \in \bigcap_{N=1}^{\infty} \sigma(S_N, S_{N+1}, \dots).$$

By "Kolmogorov's Zero-One Law" we derive

$$\mathbb{P}(\text{explosion}) \in \{0, 1\}.$$

(2b) Using Demo 2/ Problem 2b for $\lambda_n = \lambda > 0$ we have

$$\sum_{n=1}^{\infty} S_n < \infty \text{ a.s. } \iff \sum_{n=1}^{\infty} \frac{1}{\lambda} < \infty.$$

But $\sum_{n=1}^{\infty} \frac{1}{\lambda} = \infty$ so that, by (2a),

$$\sum_{n=1}^{\infty} S_n = \infty \quad \text{a.s.} \quad \text{or}$$

$$\mathbb{P}(\text{explosion}) = 0.$$