TIES435 Signal processing

Exercises #1

1. Express the sequence $x[n]=1$, $-\infty < n < \infty$, in terms of the unit step function $\mu[n]$.

2. Express the length-4 sequence $x[n]=\{1\ 3\ -2\ 4\}$, $n=0,1,2,3$, in terms of

   a) unit sample $\delta[n]$

   b) unit step function $\mu[n]$.

3.

2.5 Consider the following sequences:

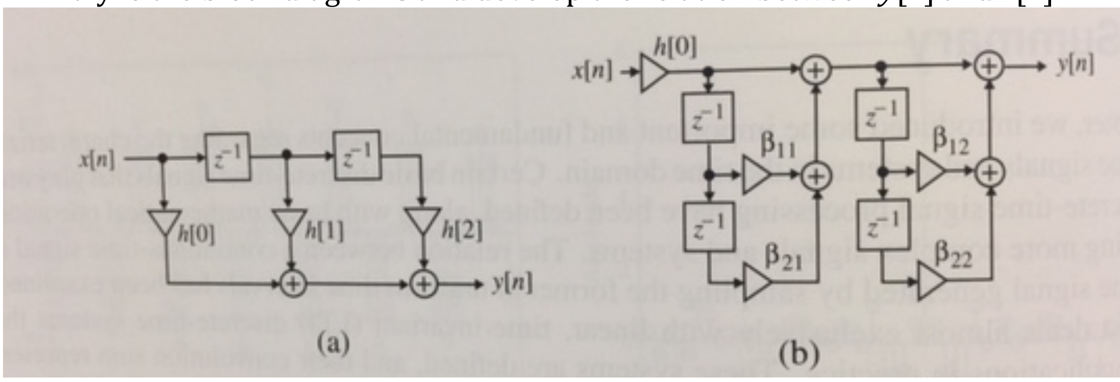
$$x[n] = \{-4\ \ 5\ \ 1\ \ -2\ \ -3\ \ 0\ \ 2\},\ -3 \le n \le 3$$

$$y[n] = \{6\ \ -3\ \ -1\ \ 0\ \ 8\ \ 7\ \ -2\},\ -1 \le n \le 5$$

$$w[n] = \{3\ \ 2\ \ 2\ \ -1\ \ 0\ \ -2\ \ 5\},\ 2 \le n \le 8.$$

The sample values of each of the above sequences outside the ranges specified are all zeros. Generate the following sequences: (a) $c[n] = x[-n+2]$, (b) $d[n] = y[-n-3]$, (c) $e[n] = w[-n]$, (d) $u[n] = x[n] + y[n-2]$, (e) $v[n] = x[n] \cdot w[n+4]$, (f) $s[n] = y[n] - w[n+4]$, and (g) $r[n] = 3.5y[n]$.

4. Analyze the block diagrams and develop the relation between $y[n]$ and $x[n]$.



5.

2.15 Which ones of the following sequences are bounded sequences?

(a) $x[n] = A\alpha^n$, where A and α are complex numbers, and $|\alpha| < 1$,

(b) $y[n] = A\alpha^n\mu[n]$, where A and α are complex numbers, and $|\alpha| < 1$,

(c) $h[n] = C\beta^n\mu[n]$, where C and β are complex numbers, and $|\beta| > 1$,

(d) $g[n] = 4\cos(\omega_a n)$, (e) $v[n] = \left(1 - \frac{1}{n^2}\right)\mu[n-1]$.

6.

2.24 Compute the energy of the following sequences:

(a) $x_a[n] = A\alpha^n\mu[n]$, $|\alpha| < 1$, (b) $x_b[n] = \frac{1}{n^2}\mu[n-1]$.

7.

2.38 The second derivative $y[n]$ of a sequence $x[n]$ at time instant n is usually approximated by

$$y[n] = x[n+1] - 2x[n] + x[n-1].$$

If $y[n]$ and $x[n]$ denote the output and input of a discrete-time system, is the system linear? Is it time-invariant?

8. Let's denote convolution by *. Show that

a) $\delta[n]*\delta[n] = \delta[n]$

b) $\delta[n]*\delta[n-m] = \delta[n-m]$

c) $\delta[n-m]*\delta[n-r] = \delta[n-m-r]$

9.

2.65 Determine the overall impulse response of the system of Figure P2.3, where the impulse responses of the component systems are: $h_1[n] = 2\delta[n-2] - 3\delta[n+1]$, $h_2[n] = \delta[n-1] + 2\delta[n+2]$, and $h_3[n] = 5\delta[n-5] + 7\delta[n-3] + 2\delta[n-1] - \delta[n] + 3\delta[n+1]$.

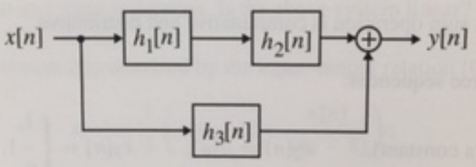


Figure P2.3

MATLAB exercises

10.

M 2.1 (a) Using Program 2_2, generate the sequences shown in Figures 2.17 and 2.18.
(b) Generate and plot the complex exponential sequence $-3.6e^{(-0.5+j\pi/4)n}$ for $0 \leq n \leq 82$ using Program 2_2.

```
% Program 2_2
% Generation of complex exponential sequence
%
a = input('Type in real exponent = ');
b = input('Type in imaginary exponent = ');
c = a + b*i;
K = input('Type in the gain constant = ');
N = input ('Type in length of sequence = ');
n = 1:N;
x = K*exp(c*n);%Generate the sequence
stem(n,real(x));%Plot the real part
xlabel('Time index n');ylabel('Amplitude');
title('Real part');
disp('PRESS RETURN for imaginary part');
pause
stem(n,imag(x));%Plot the imaginary part
xlabel('Time index n');ylabel('Amplitude');
title('Imaginary part');
```

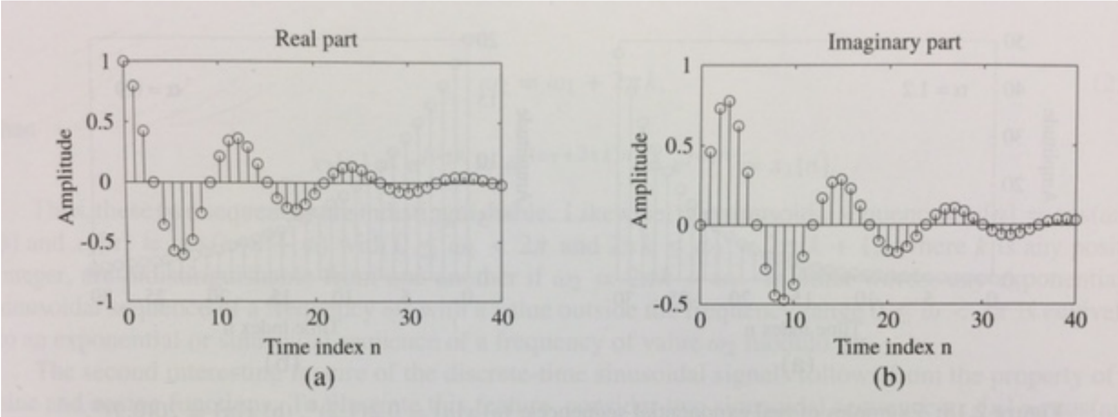


Figure 2.17: A complex exponential sequence $x[n] = e^{(-1/12+j\pi/6)n}$. (a) Real part and (b) imaginary part.

11. Generate the sequences from b) to e) using Matlab.

(a) $\tilde{x}_a[n] = e^{j0.5\pi n}$, (b) $\tilde{x}_b[n] = \sin(0.8\pi n + 0.8\pi)$, (c) $\tilde{x}_c[n] = \mathrm{Re}\left(e^{j\pi n/5}\right) + \mathrm{Im}\left(e^{j\pi n/10}\right)$,
(d) $\tilde{x}_4[n] = 3\cos(1.3\pi n) - 4\sin(0.5\pi n + 0.5\pi)$, (e) $\tilde{x}_5[n] = 5\cos(1.5\pi n + 0.75\pi) + 4\cos(0.6\pi n) - \sin(0.5\pi n)$.

12.

M 2.3 (a) Write a MATLAB program to generate a sinusoidal sequence $x[n] = A\sin(\omega_o n + \phi)$, and plot the sequence using the `stem` function. The input data specified by the user are the desired length L, amplitude A, the angular frequency ω_o, and the phase ϕ where $0 < \omega_o < \pi$ and $0 \leq \phi \leq 2\pi$. Using this program, generate the sinusoidal sequences shown in Figure 2.16.

(see the Figure 2.16 in the next page)

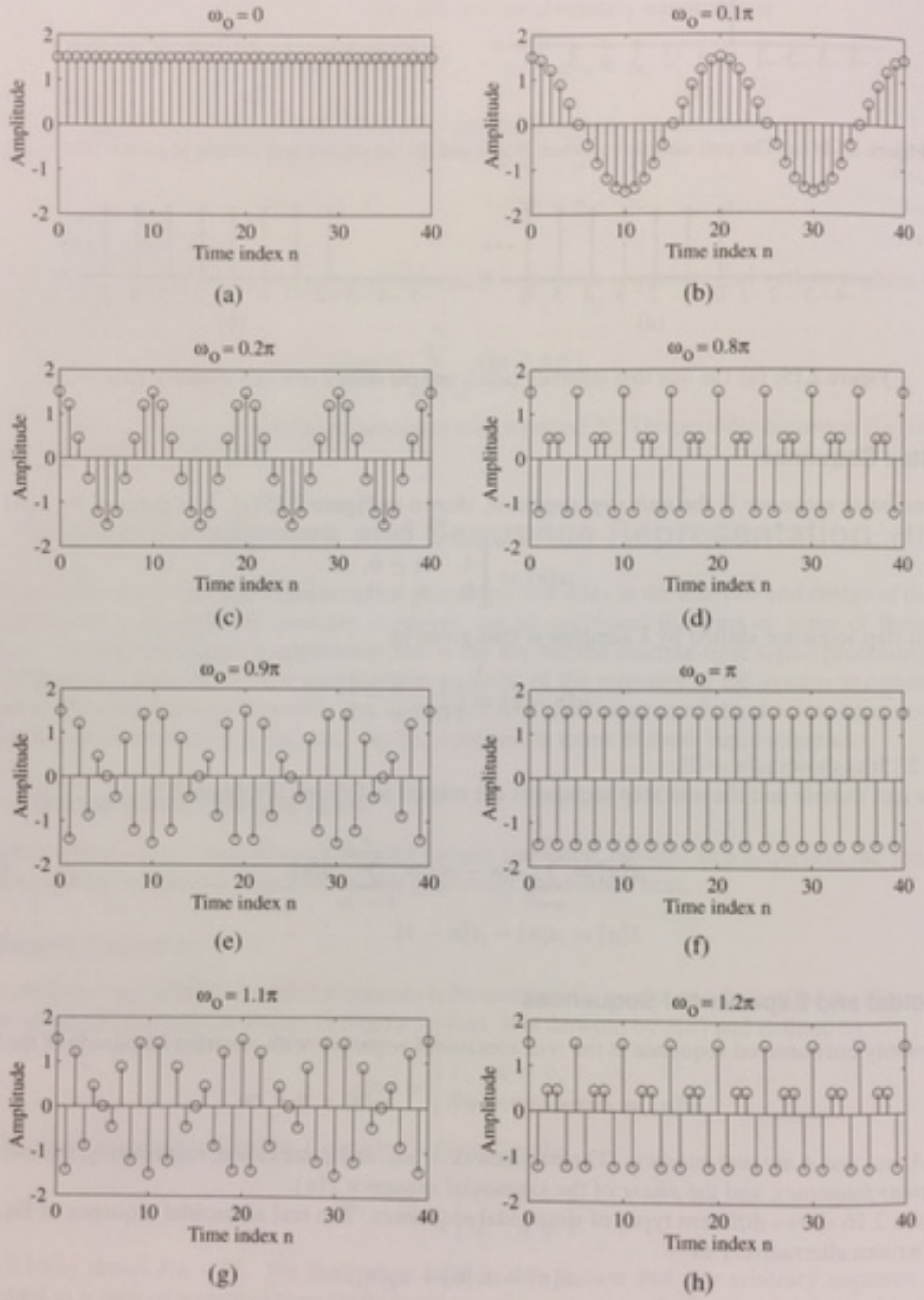


Figure 2.16: A family of sinusoidal sequences given by $x[n] = 1.5 \cos \omega_o n$: (a) $\omega_o = 0$, (b) $\omega_o = 0.1\pi$, (c) $\omega_o = 0.2\pi$, (d) $\omega_o = 0.8\pi$, (e) $\omega_o = 0.9\pi$, (f) $\omega_o = \pi$, (g) $\omega_o = 1.1\pi$, and (h) $\omega_o = 1.2\pi$.