TIES324 Signal processing

Exercises #2

1. Determine the CTFT of the following continuos-time functions:
    a) $v(t) = e^{j\Omega_0 t}$
    b) $v(t) = \cos(\Omega_0 t)$

Hint1: Euler's formula $e^{j\alpha} = \cos\alpha + j\sin\alpha$ gives also $\cos\alpha = \frac{1}{2}\left(e^{j\alpha} + e^{-j\alpha}\right)$

Hint2: Dirac's delta function property: $\int_{-\infty}^{\infty} \delta(t - t_0)x(t)dt = x(t_0)$

2. Let us assume that the CTFT of an ideal impulse $\delta(t)$ equals $\boldsymbol{\Delta}(j\Omega)=1$. Show that the CTFT of the delayed impulse $\delta(t-t_0)$ equals $\exp(-j\Omega t_0)$.

3. Show that the CTFT of $x(t) = \frac{\sin(t)}{\pi t}$ equals the following:

$$X(j\Omega) = \begin{cases} 1, & \text{when } |\Omega| \le 1 \\ 0, & \text{when } |\Omega| > 1 \end{cases}$$

Hint: Euler's formula $e^{j\alpha} = \cos\alpha + j\sin\alpha$ gives also $\sin\alpha = \frac{1}{2j}\left(e^{j\alpha} - e^{-j\alpha}\right)$

4. Derive the DTFT of the unit sample $\delta[n]$.

5. Determine the Fourier transform Y of the sequence $y[n]=\alpha^n$ for $0 \le n \le M-1$. Otherwise $y[n]=0$. Assume also $|\alpha|<1$.

6. Determine the Fourier transform of the sequence $y[n]=(n+1)\,\alpha^n\,\mu[n]$, $|\alpha|<1$.

7. Suppose we have a filter whose system equation equals

$$y[n] = \alpha_0\, x[n] + \alpha_1\, x[n-1] + \alpha_0\, x[n-2]$$

where α's are some constants. Suppose the input signal consist of a sum of two cosine sequences of angular frequencies 0.1 and 0.4. Determine the constants α so that the filter blocks the lower frequency.

8. Prove the following theorems of DTFT: a) linearity b) time-reversal c) time-shifting and d) frequency shifting, existing in the Table 3.1 (time-reversal) and Table 3.4 (others) of Lecture 3.

9.

3.35 A sequence $x[n]$ has a zero-phase DTFT $X(e^{j\omega})$ as sketched in Figure P3.3. Sketch the DTFT of the sequence $x[n]e^{-j\pi n/3}$.

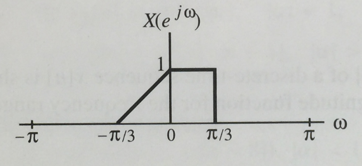

Figure P3.3

10.

3.41 Let $G_1(e^{j\omega})$ denote the discrete-time Fourier transform of the sequence $g_1[n]$ shown in Figure P3.4(a). Express the DTFTs of the remaining sequences in Figure P3.4 in terms of $G_1(e^{j\omega})$. Do not evaluate $G_1(e^{j\omega})$.

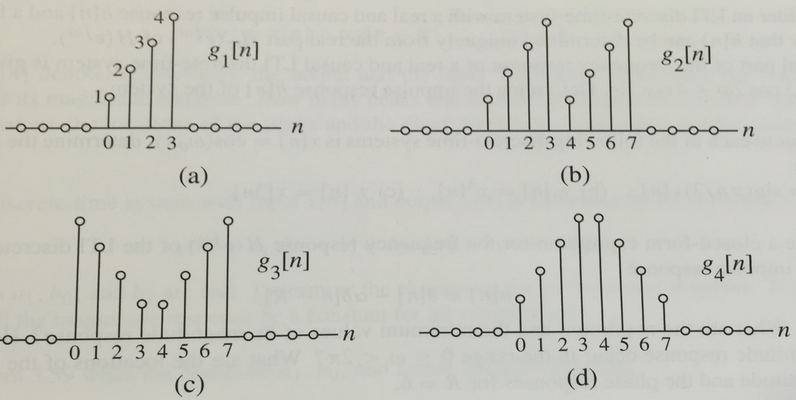

Figure P3.4

11.

EXAMPLE 3.16 Illustration of Fourier Transform Computation Using MATLAB

Program 3_1 can be employed to determine the values of the Fourier transform of a real sequence described as a rational function in $e^{-j\omega}$. The program computes the Fourier transform values at the prescribed frequency points and plots the real and imaginary parts and the magnitude and phase spectrums. It should be noted that because of the symmetry relations of the Fourier transform of a real sequence as indicated in Table 3.3, the Fourier transform is evaluated only at specified equally spaced values of ω between 0 and π.

We consider the evaluation of the following Fourier transform:

$$X(e^{j\omega}) = \frac{0.008 - 0.033e^{-j\omega} + 0.05e^{-j2\omega} - 0.033e^{-j3\omega} + 0.008e^{-j4\omega}}{1 + 2.37e^{-j\omega} + 2.7e^{-j2\omega} + 1.6e^{-j3\omega} + 0.41e^{-j4\omega}}. \quad (3.74)$$

Using Program 3_1, plot the real part, imaginary part, magnitude spectrum and phase spectrum.

```
% Program 3_1
% Discrete-Time Fourier Transform Computation
%
% Read in the desired number of frequency samples
k = input('Number of frequency points = ');
% Read in the numerator and denominator coefficients
num = input('Numerator coefficients = ');
den = input('Denominator coefficients = ');
% Compute the frequency response
w = 0:pi/(k-1):pi;
h = freqz(num, den, w);
% Plot the frequency response
subplot(2,2,1)
plot(w/pi,real(h));grid
title('Real part')
xlabel('\omega/\pi'); ylabel('Amplitude')
```

```
subplot(2,2,2)
plot(w/pi,imag(h));grid
title('Imaginary part')
xlabel('\omega/\pi'); ylabel('Amplitude')
subplot(2,2,3)
plot(w/pi,abs(h));grid
title('Magnitude Spectrum')
xlabel('\omega/\pi'); ylabel('Magnitude')
subplot(2,2,4)
plot(w/pi,angle(h));grid
title('Phase Spectrum')
xlabel('\omega/\pi'); ylabel('Phase, radians')
```

12. Use Program 3_3 to test your filter designed in task 7 of this exercise.

```
% Program 3_3
% PUT HERE YOUR FILTER COEFFICIENTS ALPHA0, ALPHA1 and ALPHA2
% FOR EXAMPLE, alpha = [2 0.1 -1.2]
alpha = [    ];
%  Set initial conditions to zero values
zi = [0 0];
% Generate the two sinusoidal sequences
n = 0:99;
x1 = cos(0.1*n);
x2 = cos(0.4*n);
%  Generate the filter output sequence
y = filter(alpha, 1, x1+x2, zi);
% Plot the input and the output sequences
plot(n,y,'r-',n,x2,'b--',n,x1,'g-.');grid
axis([0 100 -1.2 4]);
ylabel('Amplitude'); xlabel('Time index n');
legend('y[n]','x2[n]','x1[n]')
```