TIES324 Signal processing

Exercise #3

1. The Nyquist frequency of a continuous-time signal $g_a(t)$ is Ω_m. What is the Nyquist frequency of $y(t) = \int_{-\infty}^{\infty} g_a(t-\tau)\, g_a(\tau) d\tau$?

2. A 2.5 seconds long segment of a continuous-time signal is uniformly sampled (without aliasing) containing 5001 samples. What is the highest frequency component that could be present in the continuous-time signal?

3. A continuous-time signal $x_a(t)$ is composed of a linear combination of sinusoidal signals of frequencies 300Hz, 500Hz, 1.2kHz, 2.15kHz and 3.5kHz. The signal is sampled at a 2.0kHz rate and the sampled sequence is passed through an ideal lowpass filter with a cutoff frequency of 900Hz, generating a continuous-time signal $y_a(t)$. What are the frequency components present in the constructed signal $y_a(t)$?

4. The continuous-time signal
$x_a(t) = 4\sin(20\pi t) - 5\cos(24\pi t) + 3\sin(120\pi t) + 2\cos(176\pi t)$
is sampled at a 50Hz rate, generating the sequence $x[n]$. Determine the exact presentation of $x[n]$.

5. The left and right channels of an analog stereo audio signal are sampled at a 45kHz rate, with each channel then being converted into a digital bit stream using 12-bit A/D converter. Determine the combined bit rate of the two channels after sampling and digitization.

6. Determine the discrete-time signal $v[n]$ obtained by uniformly sampling a continuous-time signal $v_a(t)$ composed a weighted sum of five sinusoidal signals of frequencies 30Hz, 150Hz, 170Hz, 250Hz and 330Hz, at a sampling rate of 200Hz, as given below:
$v_a(t) =$
$6\cos(60\pi t) + 3\sin(300\pi t) + 2\cos(340\pi t) + 4\cos(500\pi t) + 10\sin(660\pi t)$.

7. Show that the transfer function $H_a(j\Omega) = \frac{a}{j\Omega + a}$ has a monotonically decreasing magnitude response with $|H_a(j0)| = 1$ and $|H_a(j\infty)| = 0$. Determine the 3-dB cutoff frequency at which the gain response is 3dB below the maximum value of 0dB at $\Omega = 0$.