Return m-files before 3.5.2013
Email the solution to :
`fysp120(at)gmail.com` Subject-line: `FNM exercise2`

1. Write your own Matlab routine, that computes integrals of the type

$$I = \int_0^\infty dx e^{-x} f(x) \approx \sum_{i=1}^{n} w_i f(x_i)$$

using the Gauss-Legendre quadrature. Here the abscissae x_i are the roots of the Laguerre polynomial (Wikipedia "Laguerre polynomials")

$$L_n(x) = \sum_{i=0}^{n} \binom{n}{i} \frac{(-1)^i}{i!} x^i \quad , \text{ where } \binom{n}{i} = \frac{n!}{i!(n-i)!} \ .$$

and the weights are (Wikipedia "Gauss-Laguerre quadrature")

$$w_i = \frac{x_i}{(n+1)^2 L_{n+1}(x_i)^2} \ .$$

Use your program to compute the integral

$$\int_0^\infty dx e^{-x} \sqrt{2x^3} \approx 1.87997 \ .$$

Try values $n \leq 40$, larger values have numerical problems.
CONTINUES ON THE NEXT PAGE!

Hints:
- The function to compute the abscissae and the weights could be

```
function [x,w]=gausslaguerre(n)
```

- **Matlab indices start from 1!**
- Find the $n+1$ coefficients $c(i)$ of the Laguerre polynomial $L_n(x)$,

$$c(n+2-i) = \binom{n}{i-1} \frac{(-1)^{i-1}}{(i-1)!} \ ,$$

so that

$$L_n(x) = \sum_{i=1}^{n+1} c(n+2-i) x^{i-1} \ . \tag{1}$$

The coefficient are in the order used in Matlab: $c(1)$ multiplies the highest power of x.
- For the factorial $n!$ you can use the function `factorial(n)`
- The roots of the Laguerre polynomial are `roots(c)`.
- Test the program so far, the roots of $L_3(x)$ (i.e. $n = 3$) are 6.289945082937476, 2.294280360279045, and 0.415774556783479
- Compute the weights. You need now the Laguerre polynomial $L_{n+1}(x)$; you can use again the sum in Eq. (1).
- The abscissae and weights for $n = 3$ should be

x	w
6.289945082937476	0.0103892565015861
2.294280360279045	0.2785177335692384
0.415774556783479	0.7110930099291742