Vector analysis

Spring 2014

Exercise 3

Recital 26.3.

1. Evaluate the work $W = \int_C d\vec{r} \cdot \vec{F}$ of the force $\vec{F}(r) = (x+yz)\hat{i} + (y+xz)\hat{j} + (z+xy)\hat{k}$ when it moves an object from the origo *O* = (0,0,0) to the point *P*= (1,1,1)
    a. along the straight line O*P*
    b. along the curve $x = t, \quad y = t^2, \quad z = t^3$
    c. along the polygonal chain *OA, AB, BP* , where *A* = (1,0,0) and *B* = (1,1,0).
    d. Calculate $\nabla \times \vec{F}$. What conclusion you can draw from this result concerning the parts a-c above?
2. Consider the position vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.
    a. Calculate $\nabla r, \quad \nabla \cdot \vec{r}, \quad \nabla \times \vec{r}$.
    b. Show that $\nabla f(r) = f'(r)\hat{r}$ ja $\nabla^2 f(r) = \nabla \cdot (\nabla f(r)) = f''(r) + 2\frac{f'(r)}{r}$.
    c. Calculate $\nabla \frac{1}{r}$ and $\nabla^2 \frac{1}{r}$.
3. The temperature $T(x, y)$ at points of the *xy*-plane is given by $T(x, y) = x^2 - 2y^2$.
    a. Draw contour diagram for *T* showing some isotherms (curves of constant temperature).
    b. In what direction should an ant at position (2,1) move if it wishes to cool off as quickly as possible?
    c. If the ant moves in that direction at speed *k* (units distance per unit time), at hwt ratedose it experience the decrease of the temperature.
    d. Along what curve through (2,-1) should the ant move in order to continue to experience maximum rate of cooling? (Adams 12.7, exercise 21).
4. Find a vector tangent to the curve of intersection of the two surfaces $x + y + z = 6$ and $x^2 + y^2 + z^2 = 14$ at the point (1,2,3). (Adams 12.7, exercise 27)
5. Calculate the divergence and curl of the vector fields
    a. $xy\hat{i} + (z^2 - 2y)\hat{j} + \cos yz\hat{k}$
    b. $e^{yz}\hat{i} + e^{xz}\hat{j} + e^{xy}\hat{k}$
    c. $\frac{x}{y}\hat{i} + \frac{y}{z}\hat{j} + \frac{z}{x}\hat{k}$
6. Derive the results
    a. $\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B})$
    b. $\nabla \cdot (\nabla \times \vec{A}) = 0$
    c. $\nabla \times (\nabla \phi) = 0$
    d. $\nabla \times (\phi \vec{A}) = (\nabla \phi) \times \vec{A} + \phi (\nabla \times \vec{A})$