**Vector analysis**
Spring 2014

Exercise 3

Recital 2.4

1. A triangle $S$ has its vertices at the points $(x,y)= (0,0)$, $(1,0)$ and $(1,1)$.

   a. Evaluate the area of the triangle by integration
   b. Calculate

$$\iint_S dS\ \frac{\sin x}{x}.$$

2. Consider the integral

$$\iint_A dA\, y^3\, \frac{e^{x^2}}{x} = \int_{y=0}^{2} \int_{x=y^2}^{4} dxdy\ y^3\, \frac{e^{x^2}}{x}.$$

   a. Draw the surface $A$.
   b. Perform the integration.

3. The region of integration $S$ is the surface $S = \{(x, y) : x + y \geq 0, y \leq 0, x \leq 1\}$.

   a. Change the variables according to $u = x + y,\ v = x$ and evaluate the corresponding Jacobian.
   b. What is the region of integration on the $(u,v)$-plane that corresponds to $S$?
   c. Evaluate

$$\iint_S dS\ x^3 \sqrt{x+y}$$

   with the change of variables given above.

4. Compute

$$\iint_S dS\ (x + 2y + 3z),$$

where $S$ is the part of the plane $2x - y + z = 3$ that is above the triangle bounded by the $x$- and $y$-axes and the line $y = 1 - 2x$.

5. Compute

$$\iint_S dS \ (x^2 + y^2 + 3z^2),$$

where $S$ is the part of the circular paraboloid $z = x^2 + y^2$ with $x^2 + y^2 \leq 9$.

6. Compute

$$\iint_S dxdy \ e^{-x^2-y^2}, \quad a^2 \leq x^2 + y^2 \leq b^2$$

with a suitable (obvious) change of variables. Draw the region $S$.

Deduce from your result the value of the integral $\int_0^\infty dx \ \mathrm{e}^{-x^2}$ .