Vector Analysis
Spring 2014

Exercise 5

Recital Wed 9.4.

1. What are the objects of the *xyz*-space whose spherical coordinates obey ("vakio"=constant)
   a) r = vakio, b) ϕ = vakio, c) θ = vakio, d) r = vakio ja ϕ = vakio,
   e) r = vakio ja θ = vakio, f) ϕ = vakio ja θ = vakio,
   g) r = vakio, ϕ = vakio ja θ = vakio?

2. Consider the point P with coordinates $(x, y, z) = (1,1,1)$. Derive in terms of the unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$ the expressions for the basis vectors $\hat{\rho}, \hat{\phi}, \hat{z}$ and $\hat{r}, \hat{\theta}, \hat{\phi}$ of the local cylindrical and spherical coordinate systems locating at this point.

3. Derive the gradient of the scalar field $u(r,\theta)$ and the divergence of the vector field $\vec{v} = v_r(r,\theta)\hat{r} + v_\theta(r,\theta)\hat{\theta}$ in polar coordinates.

4. Show that in cylindrical coordinates

$$\nabla^2 u(r,\phi,z) = \frac{\partial^2 u}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial u}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 u}{\partial \phi^2} + \frac{\partial^2 u}{\partial z^2}.$$

   Start from $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ and apply the chain rule of derivatives.

5. Derive the following expression of the divergence of a vector field $\vec{F} = F_r\hat{r} + F_\theta\hat{\theta} + F_\phi\hat{\phi}$ in spherical coordinates

$$\nabla \cdot \vec{F} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 F_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(F_\theta \sin\theta) + \frac{1}{\rho\sin\theta}\frac{\partial F_\phi}{\partial \phi}$$

   applying the chain rule (see Lecture notes p. 66).