Vector Analysis

Spring 2014

Excersize 2

Recital in 19.3.

1.
   a. Evalaute the line integral of $\vec{F} = y\hat{i} + z\hat{j} - x\hat{k}$ along the straight line from the origo to (1,1,1).
   b. Evalaute the line integral of $\vec{F} = z\hat{i} - y\hat{j} + 2x\hat{k}$ along the curve $\vec{r}(t) = t\hat{i} + t^2\hat{j} + t^3\hat{k}$ from (0,0,0) to (1,1,1).
2. Study whether the following vector fields $\vec{F}(x, y, z)$ have a potential $\phi(x, y, z)$. You can do it either by showing that a potential function does not exist or by constructing $\phi(x, y, z)$:
   a. $\vec{F} = y\hat{i} + z\hat{j} - x\hat{k}$
   b. $\vec{F} = x\hat{i} - 2y\hat{j} + 3z\hat{k}$
   c. $\vec{F} = \dfrac{x\hat{i} + y\hat{j}}{x^2 + y^2}$

3. Find out the vector field $\vec{E}(x, y, z)$, when the corresponding potential is

$$\phi(x, y, z) = [x^2 + y^2 + z^2]^{1/2}.$$

4. Calculate the tangent vector $\hat{T}$ and normal vector $\hat{N}$ of $\vec{c}(t) = [(t^3/3 - t]\,\hat{i} + t^2\,\hat{j}$ at the point t = 3. Having got the vectors check that they are perpendicular.

5. A particle moves along the curve $\vec{r}(t) = t\hat{i} + 2e^t\,\hat{j} + e^{2t}\hat{k}$.
   a. Calculate the velocity and acceleration of the particle at the moment t = 1.
   b. Calculate the distance the particle travels during t = 1 to 2.
   c. Find the equation of the tangent vector at t =0.
   d. Evaluate the curvature of the path of the particle at $\vec{r}(0)$.
6. As known from mechanics, the acceleration can be divided into a normal and tangential components:

$$\vec{a} = a_T\hat{T} + a_n\hat{N}.$$

   Show that

$$\vec{a}(t) = \frac{d^2s}{dt^2}\hat{T} + \left(\frac{ds}{dt}\right)^2 \kappa\hat{N}$$

or

$$\vec{a}(t) = \frac{dv}{dt}\hat{T} + \frac{v^2}{\rho}\hat{N},$$

where κ is the curvature of the path and ρ is the radius of the curvature.

7. A truck travelling at 80 km/h and weighting 10 000 kg is moving on an unbanked curved stretch of track. The equation of the curved section is the parabola $y = x^2 - x$ (m).
    a. What is the frictional force exerted by the wheels of the truck at the point (0,0) on the track?
    b. If the coefficient of friction for the truck is 2.5, what is the maximum speed it can achieve at the point (0,0) without going off the track?