Vector Analysis

Spring 2014

Exercise 1

Recital in We 12.3.

1. The intersection of the plane $x+y+z=1$ and the cylinder $z=x^2$ is a parabola. Give a parametric presentation for this parabola using *x* as parameter.
2. Determine the length of the parametrizised curve $\vec{r}=t^2\vec{i}+t^2\vec{j}+t^3\vec{k},\ \ 0\le t\le 1$.
3. Evaluate
   a. $\int_C (x+y)ds, \quad \vec{r}=at\vec{i}+bt\vec{j}+ct\vec{k},\ \ 0\le t\le m.$
   b. $\int_C yds, \quad \vec{r}=t^2\vec{i}+t\vec{j}+t^2\vec{k},\ \ m\ge t\ge 0.$ Here you start the integration from $t=m$. What changes if you perform the integration in the opposite direction, that is start from $t=0$?
4. Evaluate

$$\oint (x^2y^2dx + x^3ydy)$$

   counter clockwise around the square with vertices (0,0), (1,0), (1,1) and (0,1).

5. For the vector field $\vec{F}=(x^2y, y^2)$, find the value of $\int_C \vec{F}\cdot d\vec{s}$, where *C* is the portion of parabola $y=x^2$ from (0,0) to (1,1).
6. Calculate the mass of a metal string of the form $\vec{r}=3t\vec{i}+3t^2\vec{j}+2t^3\vec{k},\quad 0\le t\le 1$, assuming that the mass (in some units) per (some) unit of length in the point $\vec{r}(t)$ is $1+t$.