Vector Analysis

Spring 2014

Ex Tempore 4

Mon 17.3.

1. Evaluate the gradient ∇f of the following functions at the given points:
   a. $f(x,y) = x^2 - y^2$ at (2,-1)
   b. $f(x,y,z) = xy^2z^3$ at (3,-1,2)
   c. $f(x,y,z) = \sin x \cos y \tan z$ at $(\pi/6, \pi/4, \pi/3)$
2. Consider the function $f(x,y) = x^2 y$.
   a. Calculate the gradient ∇f .
   b. Calculate the value of the directional derivative in the direction of the vector $\vec{u} = \frac{4}{5}\hat{i} - \frac{3}{5}\hat{j}$ at (1,2).
   c. In which direction the growth of f is the strongest at (1,2)?

3. Fuction $f(x,y)$ has at the point (a,b) directional derivatives

$$\text{grad}_u f(a,b) = 3\sqrt{2},$$
$$\text{grad}_v f(a,b) = 5,$$

   where the directions *u* and *v* are given by the vectors

$$\hat{u} = (\hat{i} + \hat{j})/\sqrt{2}, \quad \hat{v} = (3\hat{i} - 4\hat{j})/5.$$

   Evaluate $\nabla f(a,b)$.