

Vector Analysis
Spring 2014

Ex Tempore 10
Wed9.4.

1. Calculate

$$I = \iiint_D yz^2 dx dy dz,$$

where D is the closed area bounded by the planes $x = 0, y = 0, z = 0$ ja $3x + 2y + 6z = 6$. Draw first D .

2. Derive for the spherical coordinate system

$$\begin{aligned}x &= r \sin \theta \cos \phi, & r &= \sqrt{x^2 + y^2 + z^2} \\y &= r \sin \theta \sin \phi, \\z &= r \cos \theta,\end{aligned}$$

the following expression of the Jacobian determinant

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin \theta.$$

3. A hollowed sphere $1 \leq r \leq 2$ centered in origo has the density (μ is constant)

$$\rho = \frac{\mu}{x^2 + y^2 + z^2}.$$

Calculate the mass

$$\iiint_D \rho dV.$$

