Vector Analysis
Spring 2014

Ex Tempore 11
Wed 23.4.

1. Calculate the surface integrals

$$\oiint \vec{F} \cdot d\vec{S} \quad \text{ja} \quad \oiint \vec{G} \cdot d\vec{S}$$

   of the vector fields $\vec{F} = x\hat{i} - 2y\hat{j} + 4z\hat{k}$ and $\vec{G} = (x^2 + y^2)\hat{i} + (y^2 - z^2)\hat{j} + z\hat{k}$ over the surface of the ball $x^2 + y^2 + z^2 = a^2$. Use Gauss's theorem, for which you should first calculate the divergences.

2. D is the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 1$. Calculate the surface integral

$$\oiint_S \left(y\hat{i} + x\hat{j} + z^2\hat{k} \right) \cdot d\vec{S}$$

   over the surface S of the region.

   Hint: Use cylindrical coordinates in the integration, $dV = \rho d\rho d\phi dz$.

3. Calculte the flux of the field $\vec{F} = (y + xz)\hat{i} + (y + yz)\hat{j} - (2x + z^2)\hat{k}$ over that part of the surface of the sphere $x^2 + y^2 + z^2 = a^2$ for which $x, y, z \geq 0$.

   Hint: Consider the closed region bounded by the part of the sphere and the coordinate planes and calculate the flux through the surface of this region by using Gauss' theorem. Then subtract the fluxes through the plane surfaces.