

Vector Analysis
Spring 2014

Ex Tempore 8
Wed 2.4.

1. Calculate in terms of cylindrical coordinates ∇u and $\nabla \cdot \vec{v}$, where

a) $u = \rho$, $\vec{v} = \rho \hat{\rho} + z \hat{z}$

b) $u = z^3 \rho$, $\vec{v} = \hat{\phi}$

2. The spherical coordinates of a point are (r, θ, ϕ) , and the position vector is

$$\vec{r} = r \sin \theta \cos \phi \hat{i} + r \sin \theta \sin \phi \hat{j} + r \cos \theta \hat{k}.$$

Derive the expressions of the basis unit vectors \hat{r} , $\hat{\theta}$ and $\hat{\phi}$ of spherical coordinate system at this point in terms of the unit vectors \hat{i} , \hat{j} and \hat{k} .
Give also the values of the corresponding scale factors h_r , h_θ and h_ϕ .

3. Calculate in spherical coordinate system ∇u and $\nabla \cdot \vec{v}$, where

a) $u = \sin \theta$, $\vec{v} = r \hat{\theta}$

b) $u = r^2 \sin \phi$, $\vec{v} = \hat{r} + \hat{\theta} + \hat{\phi}$

