① $z = x + 5iy$

a) $\mathrm{Re}(z^*) = \mathrm{Re}[(x+5iy)^*]$

$= \mathrm{Re}[(x-5iy)] = \mathrm{Re}(x) + 0 = x$

b) $\mathrm{Re}(z^2) = \mathrm{Re}[(x+5iy)(x+5iy)]$

$= \mathrm{Re}[x^2 + 5iyx + 5iyx + (+5iy)^2]$

$= \mathrm{Re}[x^2] + 0 + 0 + \mathrm{Re}[25\, i^2 y^2]$

$= x^2 + \mathrm{Re}[25(-1)\, y^2]$

$= x^2 - 25y^2$

c) $\mathrm{Im}(z^2) = \mathrm{Im}[x^2 + 5iyx + 5iyx + \overbrace{(-5iy)^2}]$

$= 0 + 10yx + 0$

d) $\mathrm{Re}(zz^*) =$

$\mathrm{Re}[(x+5iy)(x+5iy)^*] =$

$\mathrm{Re}[(x+5iy)(x-5iy)] =$

$\mathrm{Re}[x^2 - 5iyx + 5iyx - 25i^2y^2] =$

$\mathrm{Re}[x^2 + 25y^2] = x^2 + 25y^2$

e) $\mathrm{Im}(zz^*) = \mathrm{Im}[x^2 + 25y^2] = 0$

(2) $x + yi = re^{i\theta} = r(\cos\theta + i\sin\theta)$

$x = r\cos\theta$

$y = r\sin\theta$

$(x+yi)(x+yi)^* = re^{i\theta}\underbrace{r^*}_{=r}e^{-i\theta}$

$x^2 + y^2 = r^2$

$r = \sqrt{x^2+y^2}$

a) $x+yi = 6i = re^{i\theta} = r\cos\theta + ir\sin\theta$

$x=0 \quad y=6$

$\Rightarrow r = \sqrt{6^2} = 6$

$y = r\sin\theta$

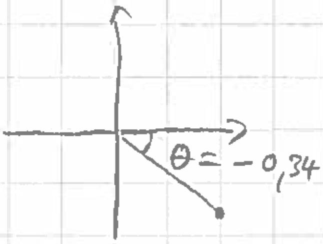
$6 = 6\sin\theta \Rightarrow \sin\theta = 1 \Rightarrow \pi/2$

$0 = 6\cos\theta \Rightarrow \cos\theta = 0 \Rightarrow \pm\pi/2$

$\Rightarrow 6i = 6e^{+i\pi/2}$

b) $x+yi = 4 - \sqrt{2}\,i$

$r = \sqrt{4^2 + (\sqrt{2})^2} = \sqrt{16+2} = \sqrt{18}$



$4 = \sqrt{18}\cos\theta \Rightarrow \theta = \pm\arccos\left(\frac{4}{\sqrt{18}}\right) + n2\pi$

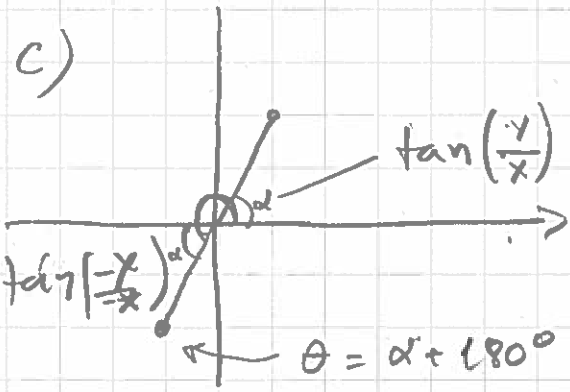
$-\sqrt{2} = \sqrt{18}\sin\theta \Rightarrow \theta = \arcsin\left(\frac{-2}{\sqrt{18}}\right) + n2\pi$

or $\theta = \pi - \arcsin\left(\frac{-2}{\sqrt{18}}\right) + n2\pi$

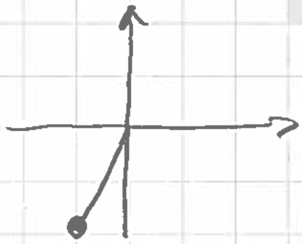
$\Rightarrow 4 - \sqrt{2}i = \sqrt{18}\exp\left[-i\arccos\left(\frac{4}{\sqrt{18}}\right)\right]$

$= \sqrt{18}\,e^{-i0{,}34}$

c)



② c) $-1-2i$



$r=\sqrt{5}$

$-1=\sqrt{5}\cos\theta$
$-2=\sqrt{5}\sin\theta$

$\theta=\pm\arccos\left(-\frac{1}{\sqrt{5}}\right)+2n\pi$

$\theta=\arcsin\left(\frac{-2}{\sqrt{5}}\right)+2n\pi$

or $\theta=\pi-\arcsin\left(\frac{-2}{\sqrt{5}}\right)+2n\pi$

$\Rightarrow -1-2i=\sqrt{5}e^{-i\arccos\left(-\frac{1}{\sqrt{5}}\right)}=\sqrt{5}e^{-2,0344i}$
$=\sqrt{5}e^{4,25i}$

d) $\pi+ei$

$r=\sqrt{\pi^2+e^2}$

$\pi=r\cos\theta$
$e=r\sin\theta$

$\Rightarrow \pi+ei=\sqrt{\pi^2+e^2}\,e^{i\,a\cos(\pi/\sqrt{\pi^2+e^2})}$

③ $h\nu = \hbar\omega = \frac{hc}{\lambda}$

a) energy conservation

$$\frac{hc}{\lambda} = K + W_e$$

photon | kinetic energy | work function (constant)

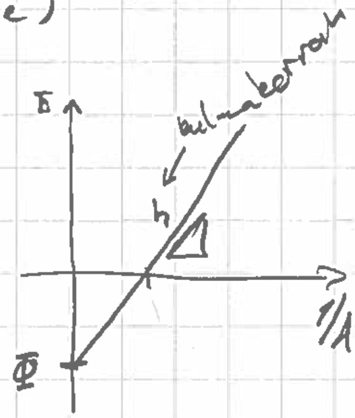
$K_1 = 2{,}935 \cdot 10^{-19}\ \mathrm{J}$ $\qquad \lambda_1 = 300\,\mathrm{nm} = 300 \cdot 10^{-9}\,\mathrm{m}$

$K_2 = 1{,}280 \cdot 10^{-19}\ \mathrm{J}$ $\qquad \lambda_2 = 400\,\mathrm{nm}$

$$\frac{hc}{\lambda_1} - \frac{hc}{\lambda_2} = K_1 - K_2 + (W_e - W_e)$$

$$h = (K_1 - K_2)\left(\frac{c}{\lambda_1} - \frac{c}{\lambda_2}\right)^{-1}$$

$$= \underline{\underline{6{,}6245 \cdot 10^{-34}\ \mathrm{Js}}}$$



c) & b) $\frac{hc}{\lambda_{min}} = 0 + W_e = \left(\frac{hc}{\lambda_1} - K_1\right)$

$$= \underline{\underline{3{,}685 \cdot 10^{-19}\ \mathrm{J}}}$$

$$\lambda_{min} = \frac{hc}{W_e} = \underline{\underline{539\,\mathrm{nm}}}$$

$$\nu = \frac{c}{\lambda} = 556\ \mathrm{THz}$$

④ $\lambda = h/p = \frac{h}{m_e v}$ $h = 6{,}626 \cdot 10^{-34}$ Js

$v = \frac{c}{100}$ $m_e = 9{,}109 \cdot 10^{-31}$ kg

$\lambda = \underline{0{,}242 \text{ nm}}$ ~ chemical bond

⑤ $\hat{p} = \frac{d^2}{dx^2}$

1) $\underbrace{\frac{d^2}{dx^2}}_{\hat{O}} \underbrace{1}_{f(x)} = 0 \Rightarrow \underbrace{0}_{\omega} \cdot \underbrace{1}_{f(x)}$ ks alla

2) $\frac{d^2}{dx^2} \sin(kx) = -k^2 \cdot \sin(kx)$ hiukkanen laatikossa

3) $\frac{d^2}{dx^2} e^{ikx} = (ik)(ik)\, e^{ikx}$ tasoaalto

$= -k^2 \cdot e^{ikx}$

$\hat{O} = \frac{d}{dx}$

$\frac{d}{dx} e^{ikx} = ik \cdot e^{ikx}$

ominaisfunktio jos

$\hat{O} f(x) = \omega f(x)$

jossa ω on vakio

(6) $\hat{A} = \frac{d}{dx}$ $\quad \hat{B} = x^2$ tehtävän asettelu hieman huono

a) Näytetään että on olemassa

$$\hat{A}^2 f(x) \neq [\hat{A} f(x)]^2$$

$$f(x) = x$$

$$\Rightarrow \hat{A}^2 f(x) = \frac{d}{dx}\frac{d}{dx} x = 0$$

$$\hat{A} f(x) = \frac{d}{dx} x = 1$$

$$[\hat{A} f(x)]^2 = 1 \neq 0$$

todistettu

b) $\hat{A}\hat{B} f(x) \neq \hat{B}\hat{A} f(x)$

$$f(x) = x$$

$$\hat{A}\hat{B} x = \frac{d}{dx}\left(x^2 x\right) = 3x^2$$

$$\hat{B}\hat{A} x = x^2 \frac{d}{dx} x = x^2 \neq 3x^2$$ todistettu

(7) $\psi(x) = \sin\left(\frac{n\pi x}{L}\right)$ kun $0 \leq x \leq L$

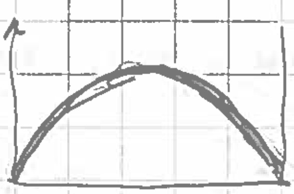
$$N = \int_0^L \sin^2\left(\frac{n\pi x}{L}\right) dx = \frac{1}{4} L \left(2 - \underbrace{\frac{\sin(2\pi n)}{n\pi}}_{0}\right)$$

taulukko/wolfram

$$= \frac{1}{2} L$$

Normitettuna

$$\tilde{\psi}(x) = \frac{1}{\sqrt{N}} \psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$



"Normitetaan jotta todennäköisyys löytää elektroni jostain 100 %".

Normitettu aaltofunktio

(7) $\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{1 \cdot \pi x}{L}\right)$

Todnäk sille että e⁻ välillä $\left[\frac{L}{2}, L\right]$

$$\int_{L/2}^{L} \left[\sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right)\right]^2 dx =$$

$$\frac{2}{L}\int_{L/2}^{L} \sin^2\left(\frac{\pi x}{L}\right) dx = \frac{2}{L}\,\frac{L}{4} = \frac{1}{2}$$

$$= 50\%$$

(8) $\varphi = \left(2 - \frac{r}{a_0}\right) e^{-r/2a_0}$

$$N = \int_0^\infty \left[\left(2 - \frac{r}{a_0}\right) e^{-r/2a_0}\right]^2 \underbrace{4\pi r^2 dr}_{\text{tilavuuselementti}}$$

$$N = 4\pi \int_0^\infty \left[\left(2 - \frac{r}{a_0}\right)^2 e^{-r/a_0}\right] r^2 dr$$

$$= 4\pi \int_0^\infty \left(4r^2 - \frac{4r^3}{a_0} + \frac{r^4}{a_0^2}\right) e^{-r/a_0}\, dr$$

$$\left[\int_0^\infty x^n e^{-ax}\, dx = \frac{n!}{a^{n+1}}\right]$$

$$= 4\pi \left[4\frac{2!}{(1/a_0)^3} - \frac{4}{a_0}\frac{3!}{(1/a_0)^4} + \frac{1}{a_0^2}\frac{4!}{(1/a_0)^5}\right]$$

$$= 4\pi \left[8a_0^3 - \cancel{24 a_0^3} + \cancel{24 a_0^3}\right]$$

$$= 32\pi a_0^3 \Rightarrow \tilde{\varphi} = \frac{1}{\sqrt{N}}\varphi$$