(1) $$\psi = \frac{2}{3} e^{ikx} + \frac{2}{3} e^{-ikx} + \frac{1}{3}\left(\frac{1}{2}\right)^{1/2} e^{3ikx} + \frac{1}{3}\left(\frac{1}{2}\right)^{1/2} e^{-3ikx}$$

c) $\hat{p}\, e^{ikx} = -i\hbar \frac{\partial}{\partial x} e^{ikx} = \hbar k\, e^{ikx}$ $\underline{\underline{\text{on}}}$

$\hat{p}\, e^{-ikx} = -\hbar k\, e^{-ikx}$ $\underline{\underline{\text{on}}}$

$\hat{p}\, e^{3ikx} = 3\hbar k\, e^{3ikx}$ $\underline{\underline{\text{on}}}$

$\hat{p}\, e^{-3ikx} = -3\hbar k\, e^{-3ikx}$ $\underline{\underline{\text{on}}}$

e) havaitsemme arvot

$p = \hbar k,\ -\hbar k,\ 3\hbar k$ ja $-3\hbar k$

ensin tarkistetaan että aaltofunktio on normitettu

$$\langle\psi|\psi\rangle = \frac{1}{2\pi}\int_0^{2\pi} \psi^* \psi\, dx$$

$$= \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\left(\frac{1}{2}\right)^{1/2}\right)^2 + \left(\frac{1}{3}\left(\frac{1}{2}\right)^{1/2}\right)^2$$

$= 1$ tässä käytetty kaavaa

$$\int_0^{2\pi} \left(e^{ikx}\right)^* e^{i\tilde{k}x}\, dx = \delta_{k,\tilde{k}}$$

$$\delta_{k,\tilde{k}} = \begin{cases} 1 & \text{jos } k = \tilde{k} \\ 0 & \text{jos } k \neq \tilde{k} \end{cases}$$

① e) todennäköisyystulkinnan mukaan todennäköisyys on komponentin normi

todennäköisyys että $p = \hbar k$

$$\left(\frac{2}{3}\right)^2 = 44,4\,\%$$

tn että $p = -\hbar k$, on $44,4\%$

tn että $p = 3\hbar k$

$$\left(\frac{1}{3}\left(\frac{1}{2}\right)^{1/2}\right)^2 = 5,6\,\%$$

tn että $p = -3\hbar k$ on $5,6\%$

b) vaikka yksittäiset komponentit ovat aaltofunktio kokonaisuudessaan ei ole, vaan se on superpositio neljästä ominaisfunktiosta

esimerkiksi

$$\langle \psi | \hat{p} | \psi \rangle = 0$$

eli liikemäärän keskimääräinen arvo on nolla

d) koska aaltofunktio ei ole ominaistila liikemäärää ei voida päätellä ilman mittausta

① a) koska $\langle \psi | \hat{p} | \psi \rangle = 0$

aalto ei etene

itseasiassa

$$\psi = \frac{2}{3}\left(e^{ikx} + e^{-ikx}\right) + \frac{1}{3}\left(\frac{1}{2}\right)^{1/2}\left(e^{3ikx} + e^{-3ikx}\right)$$

$$= \frac{4}{3}\cos(kx) + \frac{1}{3}\left(\frac{1}{2}\right)^{1/2}\cos(3kx)$$

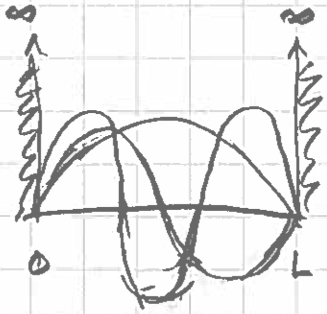
reaalinen seisova aalto

(2) a) $\psi_n = \sin\left(\frac{n\pi x}{L}\right) \qquad 0 \leq x \leq L$

S-yhtälö : $E\psi = H\psi$

tai normitettuna $E\left(\frac{\psi}{\sqrt{\langle\psi|\psi\rangle}}\right) = H\left(\frac{\psi}{\sqrt{\langle\psi|\psi\rangle}}\right)$

$\langle\psi|\psi\rangle = \int \psi^* \psi \, dx$



$H = T + V$

$= \frac{p^2}{2m} + V$

$V = \begin{cases} 0 & 0 \leq x \leq L \\ \infty & \text{muualla} \end{cases}$

$\frac{p^2}{2m} = \frac{-1}{2m}\frac{\partial^2}{\partial x^2}$

$H\psi = \frac{p^2}{2m}\psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\sin\left(\frac{n\pi x}{L}\right)$

$= \frac{-\hbar^2}{2m}\left(-\sin\left(\frac{n\pi x}{L}\right)\left(\frac{n\pi}{L}\right)^2\right)$

$= \frac{\hbar^2 n^2\pi^2}{2mL^2}\sin\left(\frac{n\pi x}{L}\right) = \frac{\hbar^2\pi^2 n^2}{2mL^2}\psi = E\psi$

$E = \frac{\hbar^2\pi^2 n^2}{2mL^2} = \frac{\hbar^2 k^2}{2m}$ jossa $k = \frac{n\pi}{L}$

$= \left(\frac{h}{2\pi}\right)^2 \frac{\pi^2 n^2}{2mL^2} = n^2 \frac{h^2}{8mL^2}$

$E\psi = H\psi$, jossa $E = n^2 \frac{h^2}{8mL^2}$

② a) $\varphi_n = \sin\left(\frac{n\pi x}{L}\right)$ $\tilde{\varphi}_n = \frac{1}{\sqrt{N}}\varphi_n$

$$N = \int_0^L \left(\sin\left(\frac{n\pi x}{L}\right)\right)^2 dx$$

$$= \frac{1}{4} L\left(2 - \frac{\sin 2\pi n}{n\pi}\right) = \frac{L}{2}$$

$$\tilde{\varphi}_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

b) $E = \frac{9h^2}{8mL^2} = (n_1^2 + n_2^2 + n_3^2)\frac{h^2}{8mL^2}$

$n_1^2 + n_2^2 + n_3^2 = 9$

n_1	n_2	n_3
2	2	1
2	1	2
1	2	2

degeneración = 3

$n_1^2 + n_2^2 + n_3^2 = 12$

n_1	n_2	n_3
2	2	2

degeneración = 1

(3) $\psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$

ortogonaalwra pos

$\langle \psi_2 | \psi_3 \rangle = 0$

$$\langle \psi_2 | \psi_3 \rangle = \frac{2}{L}\int_0^L \sin\left(\frac{2\pi x}{L}\right) \sin\left(\frac{3\pi x}{L}\right) dx$$

$$= \frac{2}{L}\int_0^L \frac{1}{2}\left[\cos\left(\frac{2\pi x}{L} - \frac{3\pi x}{L}\right) - \cos\left(\frac{2\pi x}{L} + \frac{3\pi x}{L}\right)\right] dx$$

$$= \frac{1}{L}\int_0^L \left[\cos\left(\frac{-\pi x}{L}\right) - \cos\left(\frac{5\pi x}{L}\right)\right] dx$$

$$= \frac{1}{L}\left[\frac{L\sin(-\pi)}{-\pi} - \frac{L\sin(5\pi)}{5\pi}\right]$$

$$= 0$$

(4) $$\psi = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \qquad n=1$$

$$\hat{p} = -i\hbar \frac{\partial}{\partial x}$$

$$\hat{p}\,\psi_1 = -i\hbar\frac{\partial}{\partial x}\left[\sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right)\right]$$

$$= -i\hbar\sqrt{\frac{2}{L}}\left[-\cos\left(\frac{\pi x}{L}\right)\right]\cdot\frac{\pi}{L}$$

$$= +\frac{ih}{2\pi}\sqrt{\frac{2}{L}}\,\frac{\pi}{L}\cos\left(\frac{\pi x}{L}\right)$$

$$= \frac{ih}{\sqrt{2}}\,\frac{1}{L^{3/2}}\cos\frac{\pi x}{L}$$

$$\langle\psi_1|\hat{p}|\psi_1\rangle = \frac{ih}{\sqrt{2}}\,\frac{1}{L^{3/2}}\sqrt{\frac{2}{L}}\int_0^L \sin\frac{\pi x}{L}\cos\frac{\pi x}{L}\,dx$$

$$= \frac{ih}{L^2}\int_0^L \frac{1}{2}\left[\sin\left(\frac{\pi x}{L}+\frac{\pi x}{L}\right) + \underbrace{\sin\left(\frac{\pi x}{L}-\frac{\pi x}{L}\right)}_{0}\right]dx$$

$$= \frac{ih}{2L^2}\int_0^L \sin\left(\frac{2\pi x}{L}\right)dx$$

$$= \underline{\underline{0}}$$

(4) $$\hat{p}^2 \psi_1 = -\hbar^2 \frac{\partial^2}{\partial x^2} \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right)$$

$$= -\hbar^2 \sqrt{\frac{2}{L}} \left(-\sin \frac{\pi x}{L}\right) \left(\frac{\pi}{L}\right)^2 = \left(\frac{\hbar \pi}{L}\right)^2 \psi_1$$

$$\langle \psi_1 | \hat{p}^2 | \psi_1 \rangle = \left(\frac{\hbar \pi}{L}\right)^2 \underbrace{\langle \psi_1 | \psi_1 \rangle}_{\text{1 koska normitettu}}$$

$$= \hbar^2 \frac{\pi^2}{L^2}$$

(5) (Atkins 8 ed chapter 9 example 9,2)

$\psi = B e^{-Kx}$ $\quad K = \sqrt{2 m_e (V-E)/\hbar^2}$

$V - E = 2\,eV$

oletetaan että kärjen etäisyys näytteestä on vähintään 0,5 nm

$KL = \sqrt{2 m_e (V-E)/\hbar^2}\; 0{,}5\,nm = 3{,}62$

aika iso (vaikkei $\gg 1$) käytetään yhtälöä

$$T \approx 16\, \varepsilon (1-\varepsilon)\, e^{-2KL}$$

Virta on verrannollinen T:hen.

5

$$\frac{T_1}{T_2} = \left(1 + 2\text{‰}\right) = 1{,}02$$

$$\frac{16\,\varepsilon(1-\varepsilon)\,e^{-2KL_1}}{16\,\varepsilon\,(1-\varepsilon)\,e^{-2KL_2}} = 1{,}02$$

$$\Rightarrow \quad e^{-2K(L_1-L_2)} = 1{,}02$$

$$-2K\,(L_1-L_2) = \ln 1{,}02$$

$$L_1 - L_2 = \frac{\ln 1{,}02}{-2K}$$

$$= \frac{\ln 1{,}02}{-2\cdot 7{,}25\cdot 10^{9}/m}$$

$$L_2 - L_1 = 1{,}366\cdot 10^{-12}\,m$$

$$= \underline{\underline{1{,}4\ pm}}$$