

MATEMATIIKAN PERUSKURSSI

Exercise 1 26.1.2017

The Exercise-sessions are on Thursday 26.1.2017. If you cannot attend, the exercises can also be returned until 14.00 Thursday 26.1.2017 either to juha.m.ylinen@jyu.fi, or directly to my office MaD308.

1. What are the first five terms of the sequences $(2 \cdot i - 5)_{i=1}^{23}$ and $((-1)^i(i^2 - 4 \cdot i + 4))_{i=1}^{\infty}$?
2. Are the following sequences equal?
 - (a) 1, 2, 1, 2, 1, 2 and 1, 2, 1, 2, 1, 2, ...
 - (b) 1, 1, 2, 1, 1, 2, 1, 1, 2 and 1, 2, 1, 1, 2, 1, 1, 2, 1
 - (c) 1, 4, $\sqrt{2}$, 0.5, 0, $\cos(\pi)$, 12 and $2^0, 2^2, 2^{\frac{1}{2}}, 2^{-1}, \log(1), -1, 12$
3. Are the following sequences arithmetic/geometric? If a sequence is arithmetic/geometric, then how much is the difference of two consecutive terms d , or the ratio of two consecutive terms q ?
 - (a) 1, 2, 3, 4, 5, 6, 7, 8, 9
 - (b) 4, 2, 1, $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$
 - (c) 2, 4, 6, 8, 12, 14
4. Let $(a_i)_{i=1}^{\infty}$ an arithmetic sequence with $a_1 = 8$ and $a_{13} = 92$. How much is the difference of two consecutive terms? Also, compute a_{25} .
5. Assume that $\sum_{i=1}^{10} a_i = 161$ and $\sum_{i=1}^{10} b_i = -5$. Compute the sum $\sum_{i=1}^{10} (a_i + 8b_{11-i} - 2 \cdot i)$.
6. Assume that the sums $\sum_{i=1}^n a_i$ and $\sum_{i=1}^n b_i$ are arithmetic. Is the sum $\sum_{i=1}^n (a_i + b_i)$ also arithmetic? Justify your answer.
7. You take a loan of 12000€ with a 3 % yearly interest rate. You pay the loan back using an even principal payment schedule¹ monthly for 10 years. How much do you pay interest altogether?

¹This means that at each payment you pay the same principal, and you also pay the interest that has accumulated after your last payment.

Let $x, y, p > 0$. We say, that y is $p\%$ larger than x if

$$y = \left(1 + \frac{p}{100}\right) x.$$

Let $x, y > 0$ and $0 < p \leq 100$. We say, that y is $p\%$ smaller than x if

$$y = \left(1 - \frac{p}{100}\right) x.$$

8. If y is $p\%$ larger than x , then x is $q\%$ smaller than y . Solve q .

9. You and your friend have a summer job that pays each of you 1000€/month. After the first week the company accidentally lowers your pay by 10%,² and raises your friend's salary by 10%. The company notices its mistake the following day, and raises your salary by 10%, and lowers your friend's salary by 10%. How much is your salary now? How about your friend's salary?

10. The company hires you as a permanent employee. You manage to negotiate a 4% raise. The company would like for the raise to become effective after a year, but you want the raise to become effective right away. However, the company accepts a compromise, where you get a raise of 2% now, and another raise of 2% after a year. How many % is your salary greater after the raises, compared to what it was before the raises?

(11*). Assume that $p + q = 4$. Prove, that if you first get a raise of $p\%$, and then a raise of $q\%$, your salary after the raises is largest with the choice $p = q = 2$.

(12*). You invest $x\text{€}$ into stock. With probability 0.95 the course of the stock is raised by 10% in a year, and with probability 0.05 the course of the stock goes down by 90%. What is the *expected value* of your investment, i.e. how much do you have in average after one year? Which is more profitable in average, investing into the stock, or depositing the $x\text{€}$ into a bank account that gives you a 2% yearly interest?

Bonusquestion (does not affect the exercise points): Which would you choose, the investment or the bank account, when:

(a) $x = 100\text{€}$?

(b) $x = 100\text{€} +$ everything you will earn in the next 15 years?

²That means, your new salary is 10% smaller than your old salary.