Matematiikan peruskurssi
Exercise 10
30.3.2017

(1*). Fill in the Webropol-questionnaire related to this course.
(NOTE: The questionnaire opens on 30.3.2017, but you don't have to fill it in before the exercise-session. However, do it before the questionnaire closes, which is on 20.4.2017 at 01.00)

2. Solve the separable differential equation

$$y'(x) = 2xy(x) - y(x).$$

3. Solve the differential equation

$$y'(x) = \frac{2x}{y(x)^2}.$$

4. Solve the initial value problem

$$\begin{cases} y'(x) = xe^{x^2 - \ln(y(x)^2)} \\ y(0) = 1 \end{cases}$$

5. Prove that $y_1(x) = 2x - 1$ is a solution to the linear differential equation $y'(x) + y(x) = 2x + 1$. Find the general solution by first solving the related homogeneous differential equation.

6. The function $y_1(x) = xe^{-x^2}$ is a solution to the linear differential equation $y'(x) + 2xy(x) = e^{-x^2}$. Find the general solution by first solving the related homogeneous differential equation.

7. Solve the linear differential equation $y'(x) + \sin(x)y(x) = 2\sin(x)$ by applying the formula $y(x) = \left(\int e^{F(x)}g(x)\mathrm{d}x + C\right) e^{-F(x)}$.

8. Solve the linear differential equation $y'(x)+(1+2x)y(x) = (1+2x)(x+x^2)$ by applying the formula $y(x) = \left(\int e^{F(x)}g(x)\mathrm{d}x + C\right) e^{-F(x)}$.

9. Solve the differential equation

$$y'(x) - 5y(x) = 4e^{3x}.$$

(Hint: The equation has a solution of form $y(x) = Ke^{3x}$, where $K \in \mathbb{R}$. Solve K, and solve the homogeneous equation. Alternatively you may use the solution formula)

10. Solve the initial value problem

$$\begin{cases} y'(x) = 6 - y(x), \\ y(\log(2)) = 3 \end{cases}$$

(11*). Solve the initial value problems

$$(a) \begin{cases} y'(x) = \frac{x}{y(x)} \\ y(2) = -4 \end{cases} \qquad (b) \begin{cases} y'(x) = \frac{x}{y(x)} \\ y(2) = 1 \end{cases} \qquad (c) \begin{cases} y'(x) = \frac{x}{y(x)} \\ y(2) = 2 \end{cases}$$

(12*). Solve the initial value problem

$$\begin{cases} y'(x) = \frac{(y(x)^2 - 1)}{2} \\ y(1) = 0 \end{cases}$$

(Hint: Exercise 7 / (11*) helps)

(13*). Solve the differential equation

$$y'(x) - 2y(x) = 4x^2 - 2x - 3.$$

(Hint: The equation has a solution of form $y(x) = Ax^2 + Bx + C$. Solve A, B, C, and solve the homogeneous equation. Or use the solution formula)

(14*). Solve the initial value problem:

$$\begin{cases} y'(x) = \frac{2x^3 e^{x^4}}{y(x) e^{y(x)^2}} \\ y(1) = 1 \end{cases}$$

(15*). Solve the initial value problem

$$\begin{cases} y'(x) - y(x) = x \\ y(\log(3)) = 5 - \log(3) \end{cases}$$

(Hint: The differential equation is linear. Find the general solution, then use the initial value)

(16*). Solve the initial value problem

$$\begin{cases} y'(x) + \frac{y(x)}{x \log(x)} = |\log(x)| \\ y(e) = 2e \end{cases}$$

(Hint: This is also a linear differential equation)

(17*). Assume that the functions $y_1(x)$ and $y_2(x)$ are solutions to the differential equation

$$y'(x) + f(x)y(x) = 0, \tag{1}$$

where f is some function. Prove that also the functions $y_1(x) + y_2(x)$ and $ay_1(x)$ for all $a \in \mathbb{R}$ are solutions to differential equation (1).[1]

(18*). Assume that the functions $y_0(x)$ and $y_1(x)$ are solutions to the differential equation (1), and assume that $y_0(x) \neq 0$ and $y_1(x) \neq 0$ for all $x \in \mathbb{R}$. Prove that there exists $C \in \mathbb{R}$ such that $y_0(x) = Cy_1(x)$.[2]
(Hint: Differentiate the function $\frac{y_0(x)}{y_1(x)}$)

[1] This means that the solutions to equation (1) form a linear space.
[2] This essentially shows that the solutions to equation (1) form a **one-dimensional** linear space.