

Matematiikan peruskurssi

Exercise 2

Thursday 2.2.2017

1. Compute the sum $\sum_{i=1}^n a_i$ up to two digits, when

(a) $a_i = 3 \cdot 4^{i-1}$ and $n = 5$

(b) $a_i = 3 \cdot \left(\frac{2}{3}\right)^i$ and $n = 15$

2. Let $(a_i)_{i=1}^{\infty}$ a geometric sequence, with $a_2 = 243$ and $a_7 = 1$. How much is the ratio of two consecutive terms? How about the sum of the first six terms?

3. At the beginning of the year you deposit 20000€ into a bank account. How much do you have in the account at the end of the year, if the interest rate of the account is

(a) 1% per year?

(b) $\frac{1}{12}$ % per month?

(c) $\frac{1}{52}$ % per week¹?

(d) $\frac{1}{360}$ % per day²?

Additional question (does not affect your points): What if the bank pays an interest of $\frac{1}{n}$ % every $\frac{1}{n}$ years, and you let n tend to infinity? (Hint: Find the definition of *Neper number*)

4. You get a loan of 50000€ for 10 years with a fixed 8% yearly interest, and pay the loan back using an *even total payment schedule*³. How much is the total payment, and how much do you pay interest altogether, when you pay the loan back

(a) once a year?

(b) once a month?⁴

¹A year equals 52 weeks.

²A year equals 360 days.

³This means, that the amount you pay is the same in each round.

⁴By simple interest rate, the interest rate in this case is $\frac{8}{12}$ % per month.

5. (continuation of 4) Compute the end value of your loan in assignment 4, i.e. prolong your payments to the end of the loan using 8% yearly interest, when you pay the loan back

- (a) once a year,
- (b) once a month.⁵

Also, compute the current value of the loan, i.e. discount the end value of the loan to the moment when you take the loan, using an 8% yearly interest.

6. (continuation of 4) How much interest would you pay from the loans of assignment 4, if you used an even *principal* payment schedule instead?

7. (continuation of 4) Tony Soprano approaches you, and offers you an alternative so called *Bullet-loan*: You pay back the whole principal of the loan after 10 years, and until that you only pay interest⁶, and the interest is 1% **per week**. How much interest altogether do you pay on this loan?

8. You anticipate to finish your Master's degree in 4 years, and plan to have a rather large party at that time. You estimate that the party costs about 10000€, and start saving at the last day of this month (so your first deposit gathers interest altogether 47 months). How much should you deposit monthly to a bank account to have 10000€ after 4 years, if

- (a) the account gives an interest of $\frac{1}{12}\%$ per month?
- (b) the account gives an interest of 1% per year, and the interest on part-year is calculated using simple interest rate?⁷

(9*). A bank offers you two choices for an interest rate on your savings account. Either

- (a) 8%, and the interest is added to the account once a year, or
- (b) $r\%$, and the interest is added to the account once a month.

⁵Prolong first the payments of one year to the end of that year using simple interest rate.

⁶Tony uses the term "vig".

⁷For example, the money that stands on the account for 11 months, gather interest $\frac{11}{12}\%$.

Find such r , that gathers exactly the same amount during one year to bank account (b), as you do for the bank account (a). Calculate the payments corresponding to your interest rate ($r\%$ per month) in 4(b). Also, calculate the end value of this loan, and the current value, again using the interest rate $r\%$ per month.