

Matematiikan peruskurssi

Exercise 3

9.2.2017

In exercises 1-6 we use the matrices

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 5 & 1 \\ 4 & 0 & 1 \\ 0 & 5 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix},$$

$$D = \begin{bmatrix} 1 & 0 & 1 & 4 \\ 0 & 2 & 0 & 4 \\ 3 & 0 & 0 & 1 \end{bmatrix}, \quad E = [4 \ 5 \ 1 \ 1 \ 2] \quad \text{ja} \quad F = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 2 \\ 2 \end{bmatrix}$$

1. What are the dimensions of the matrices A , B , C , D , E ja F , i.e. how many rows and columns do they have? Are some of these matrices row- column- or squarematrices? How about diagonalmatrices? Or symmetric?

2. Compute

(a) $A + B$

(b) $B - C$

3. Compute $C - 5A - 5B$ ja $D + A$, if possible.

4. Compute AD .

5. Compute

(a) D^T

(b) A^T

(c) $D^T A^T$ (Hint: rules of transpose computation might help)

6. Compute

(a) EF

(b) FE

7. Let

$$A = \begin{bmatrix} 3 & 2 & 0 \\ 4 & 0 & 1 \\ 0 & -3 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & \frac{1}{5} & 0 \\ 3 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 4 & 1 & 0 \\ 15 & -1 & 0 \\ 5 & 5 & 4 \end{bmatrix}.$$

Compute $5AB - AC$ ja $5BA - CA$. (Hint: you may use the rules of matrix multiplication and addition).

8. Compute the determinants

(a) $\det \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$

(b) $\det \begin{bmatrix} 2 & 1 & 1 \\ 0 & 3 & 4 \\ -2 & 1 & 2 \end{bmatrix}$.

9. Let $C = \begin{bmatrix} c_{11} & 0 & \cdots & 0 \\ 0 & c_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & c_{nn} \end{bmatrix}$.

Compute (or guess in part (b)) the determinant of the diagonal matrix C , $\det C$, when

(a) $n = 3$

(b) $n > 3$

(10*). Find 2×2 -matrices A , B , C , and D for which it holds:

(a) $AB \neq BA$,

(b) $CD = 0$, but $C \neq 0$ and $D \neq 0$ (note that here "0" stands for the 2×2 -matrix whose all terms are zero).