Matematiikan peruskurssi
Exercise 4
16.2.2017

Let

$$A = \begin{bmatrix} 6 & 12 & 3 \\ 0 & -1 & 1 \\ 1 & 3 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 3 & 4 \\ -2 & 1 & 2 \end{bmatrix},$$

$$D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \quad \text{ja} \quad E = \begin{bmatrix} -4 & 2 & 0 & 4 \\ 2 & 0 & 4 & 4 \\ 6 & -2 & 2 & -2 \\ 9 & -2 & 2 & 1 \end{bmatrix},$$

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1. Compute $\det E$. (Remember the row- and column-manipulations that don't change the determinant)

2. Compute $\det D$, $\det(DD^T)$, and $\det(D^T D)$, if possible. Compute also the inverse matrices of $D, DD^T, D^T D$, if possible.

3. Compute A^{-1}.

4. Compute B^{-1}.

5. Compute $(AB)^{-1}$.
(Hint: rules for computing the inverse matrix might help)

6. Compute $\det(EE^T)$ and $\det(E^T E)$.
(Hint: rules of calculus for determinant might help)

7. Using the inverse matrix, solve the system of linear equations

$$\begin{cases} 6x_1 & +12x_2 & +3x_3 & = 3 \\ & -x_2 & +x_3 & = 1 \\ x_1 & +3x_2 & & = 1 \end{cases}$$

8. Solve using Gauss-Jordan

$$\begin{cases} x_1 + x_3 + x_4 = 3 \\ x_2 + 3x_4 = 6 \\ 3x_1 + x_2 = 0 \\ 5x_1 + 3x_2 + x_4 = -1 \end{cases}$$

9. Solve

$$\begin{cases} x_1 & & +x_3 & =1 \\ 3x_1 & +2x_2 & +x_3 & =8 \\ -x_1 & & +3x_3 & =0 \\ & 2x_2 & +4x_3 & =6 \end{cases}$$

10. Solve

$$\begin{cases} 5x_1 + & 3x_2 + 3x_3 & +2x_4 = 17 \\ & 2x_2 + 2x_3 & = 4 \\ 6x_1 & & +6x_4 = 24 \end{cases}$$

(11*). Let

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}.$$

Compute

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = A \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

and

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = A \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Compute the area of the parallelogram with vertices $(0,0), (x_1, x_2), (y_1, y_2)$, and (x_1+y_1, x_2+y_2). Compare this area to the number $\det A$. What happens if

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}?$$