Matematiikan peruskurssi
Exercise 6
2.3.2017

1. Calculate the derivative of

(a) $f(x) = 3x^3 - 4x^2 + 125$

(b) $g(x) = 2\sqrt{x}$

and evaluate the derivative at $x = 1$.

2. Calculate the derivatives of

$$(a)\ x^5 - 6x, \quad (b)\ 2\cos x + \log(x), \quad (c)\ x\sqrt{x}$$

3. Calculate the derivatives of

$$(a)\ 2x^2e^x, \quad (b)\ \frac{x^2+2x}{x+2}, \quad (c)\ x\log(x) - x$$

4. Calculate the derivatives of

$$(a)\ e^{-x^2}, \quad (b)\ \log(x^8+1), \quad (c)\ \sqrt{\cos(x-8)}$$

5. Calculate the derivatives of

$$(a)\ \log(x^2+1) + x, \quad (b)\ \frac{x^4-2x^2}{x^3+18x}, \quad (c)\ \frac{x^4-x^2+1}{x^4+1}$$

6. Let $f(x) = x^4 - 3x - 1$. What is the slope of the tangent line that is drawn to the graph of the function f at point $(1, f(1))$?

7. By using the definition of derivative (i.e. using the difference quotient) compute the derivative of the function $f(x) = (3x+2)^2$ at point -1, that is, compute $f'(-1)$. Do you get the same result by applying rules of derivative?

8. Differentiate the function $f^{-1} : \mathbb{R} \to \mathbb{R}$, when

(a) $f(x) = x^5$

(b) $f(x) = 1 - 2x$

(Also, in both items, you may compute $f'(x) \cdot (f^{-1})'(f(x))$.)

9. Calculate $y'(1)$, when $y = y(x)$ satisfies the equation:

$$y^2 - x^2 = 4.$$

10. Your dog gets loose at the center of a field, that is, at point $(0, 0)$ of the plane. You remember an episode of "avara luonto", where it was said, that a loose dog runs (disregarding everything around it) according to the following equation:

$$(x^2 + y^2)^2 - 5x^2 + y^2 = 0.$$

(a) Solve what y equals, when $x = 1$.
(Hint: You get a 4th degree equation, that can be solved using the formula for 2nd degree equations. You should get 2 solutions)

(b) You reason that it is best to catch the dog by moving to the same direction as the tangent line of the dogs running curve. Find the slopes of the tangent lines when $x = 1$.
(Hint: Implicit differentiation; in item (a) you computed the values $y_1(1)$ and $y_2(1)$).

(11*). Define function f such that its derivative is $f'(x) = xe^x$.
(Hint: first see what happens when you differentiate xe^x)

(12*). Define function f such that its derivative is $f'(x) = \frac{4x^3+8x}{x^4+4x^2+6}$.
(Hint: the nominator is the derivative of the denominator, maybe this helps?)

(13*). Prove that the function $f : [-\frac{\pi}{2}, \frac{\pi}{2}] \to [-1, 1]$, $f(x) = \sin x$ has an inverse function $f^{-1} : [-1, 1] \to [-\frac{\pi}{2}, \frac{\pi}{2}]$.
(Hint: Don't try to find an expression for the inverse function.)

(14*). Assume that for $f : A \to B$ there exists an inverse function $f^{-1} : B \to A$. Justify the following formula by using chain rule of differentiation:

$$(f^{-1})'(f(x)) = \frac{1}{f'(x)}, \quad \text{when } x \in A.$$

(Hint: When you differentiate the expression $f^{-1}(f(x))$ using chain rule, or using the knowledge $f^{-1}(f(x)) = x$, you should get the same result.)

(15*). For the function $f^{-1} : [-1, 1] \to [-\frac{\pi}{2}, \frac{\pi}{2}]$ defined in exercise (13*), compute the derivative at $x = \frac{1}{2} = \sin(\frac{\pi}{3})$, i.e. compute $(f^{-1})'(\frac{1}{2})$.