Matematiikan peruskurssi MATY020

A. General information about the exam.

- Remember to sign up to the exam in Korppi.
- Exams are on 19.4.2017 and 3.5.2017.
- Both exams are on Wednesday, start approx. at 8.00, and end at 12.00. You must stay in the exam at least for 30 minutes.
- The exam takes place in Mattilanniemi. The room is mentioned in the entrances to MaA- and MaD-buildings at the day of the exam, and also the day before.
- You may have a function calculator, but not a symbolic calculator with you. There are a few calculators to borrow in the room of the exam.
- Besides the calculator, you may take with you to the exam: writing equipment and a proof of identity (and possibly some snacks/drinks). When you return your answer sheet to the supervisor of the exam, you must prove your identity (student-card or passport or driver's license is fine).
- The formulas you need in the exam are at the flip-side of the exam paper.

B. Instructions on how to answer.

(1) Write your full name and date of birth in your answer paper(s) at the appropriate location.

(2) Read the questions carefully.

(3) Almost always, the *answer* consists of more than just the correct *result*. Also include the deduction you made, and it is wise to include a sufficient number of steps to your calculations. Then:

- the person grading the exams can see that you understood the subject, and did not just guess,
- if you made an error at some point, then the person grading the exams can see *what kind of* mistake you made. If, for example, you say that: $D(\frac{1}{x}) = -\frac{1}{x^2}$, and afterwards forget the "$-$"sign in front of $\frac{1}{x^2}$, you make a smaller mistake than without mentioning $D(\frac{1}{x}) = -\frac{1}{x^2}$. Being sloppy is not as bad as not understanding.

This being said, only write the deductions/calculations/formulas that are *relevant to the question in hand*.

(4) Being thorough is also a good idea: when you have a solution to a question, use a moment to investigate your solution. If you

notice that your solution is not possible (for example the derivative of the function $\frac{1}{x}$ should be negative, since the function itself is decreasing), then look for an error in your calculations/deductions if you have time. If you cannot find the mistake, or have no time to look for it, then write in your answer paper that the solution cannot be correct ("this is the solution I have, but it cannot be true because..."). By doing this you show that you understood the subject, even though there might be an error in your calculations.

(5) Even if you cannot solve an assignment completely, try. If you think you know something related to the assignment, then write it in the answer paper. Also, you can explain how you think the assignment should be solved, even if you cannot actually do it yourself (for example, you know that you can see where a function is increasing/decreasing by investigating the derivative, but you don't know how to derivate the function).

(6) Even if you don't manage to solve any assignment, still leave a (blank) paper with your name and date of birth to the supervisor of the exam. This way we don't have to look for "lost" answer papers.

C. Contents of this course

The essential contents of this course, i.e. the things you are supposed to learn during the course[1]:

(1) Sequences and finance

- Recognizing arithmetic and geometric sequence and sum, using the formula of the general term of the sequence, and using the formula to calculate these sums.
- Calculating the cumulated and original capital, i.e. discounting and prolonging.
- Calculating the common end-value and original value of periodic payments.
- Even total payment schedule: understanding the concept, and calculating the total payment.

(2) Matrices

- Basic computations (addition and multiplication, and multiplying by a real number).
- Calculating the transpose.
- Calculating the determinant, and the connection to the inverse matrix.
- The concept of inverse matrix, and calculating the inverse matrix (Gauss-Jordan).
- Solving a system of linear equations (Gauss-Jordan/using inverse matrix).

[1]So, when reading for the exam, at least glance through these subjects.

(3) Analysis

- Domain and range of function.
- Concept of function composition, and forming the expression.
- Inverse function: what kind of function has an inverse function, and how to form the expression. What are the domain and range of the inverse function.
- Calculating derivatives using the rules of differentiation.
- The connection between the derivative and tangent lines drawn to the graph of the function.
- The connection between the derivative and the monotonicity of the function.
- Using the derivative to find extreme values, and to determine their type (local/global minimum/maximum).
- Implicit differentiation.
- Approximating the values of the function near a given point using the derivative/tangent line.
- Computing indefinite integrals using the rules of integral calculus.
- Definite integral, computation using indefinite integral.
- Integration by parts.
- Integration by substitution.
- Calculating the volume of the area between graphs of two functions.
- Cumulative function.
- Improper integrals.

(4) Differential equations

- Recognizing the type of the differential equation (solvable by integrating/separable/linear).
- Solving the differential equation using correct solution formula.
- Solving an initial value problem, that is, finding the solution that satisfies the given additional condition.

D. Example on how to investigate your solution.

(a) Periodic payments: if you deposit 100 euro to a bank every year, then during 10 years you have made 10 deposits. Since you get some interest on your deposits, after 10 years your account should hold **more than** $100 * 10 = 1000$ euro.

(b) Periodic payments: you take an even total payment loan for 10 years, with the amount of loan 1000 euro, and you pay the loan back once a year. Since you have to also pay some interest, altogether you pay back more than 1000 euro. You pay the loan back in 10 payments, so the total payment should be **greater than**

$\frac{1000}{10} = 100$ euro. If you pay the loan back every 6 months, there are 20 payments, so the total payment must be greater than $\frac{1000}{20} = 50$ euro.

(c) Inverse matrix: When you have solved A^{-1}, the inverse matrix of A, check that you really have $AA^{-1} = I$ (or $A^{-1}A = I$).

(d) System of linear equations: When you have solved the extended matrix with Gauss-Jordan to a simple form, then give your solution as: "$x_1 = a$, $x_2 = b$,..." Check that your solution really satisfies the given system of linear equations.

(e) Inverse function: Sketch the graph of the original function. The graph of the inverse function should be this graph "inverted": Compare the graphs of the functions h and h^{-1} in exercises H5/T7 and H5/T8 (the graph of the function h^{-1} can be found in Koppa).

(f) Inverse function: If you solve the inverse function by manipulating the equation $y = f(x)$ to a form where the variable x is on one side, and on the other side you have an expression of the variable y, then this expression of y equals $f^{-1}(y)$. Check: does it hold $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$ for all appropriate x.

(g) Differentiation: If the derivative of an increasing function is negative, there is a mistake somewhere.

(h) Extremes: Sketch the graph of the function based on your knowledge of the derivative of the function (just as in exercise H7/T1). Does the graph look as you would expect, when you compare it to the expression of the function.

(i) Integration: The solution can always be checked by differentiating it.

(j) Definite integral: If you can sketch the graph of the function, then do so. From the picture you can estimate the size of the definite integral, by looking at the corresponding area and its volume. If you calculated the definite integral to be 1, and from the graph you estimate that the corresponding area has a volume of approximately 4, then you may want to recheck your calculations.

(k) Definite integral: If the function to be integrated is positive on the interval of integration (respectively negative), then the value of the definite integral must also be positive (respectively negative).

(l) Solution of differential equation: The solution can always be checked by differentiating it. If for example you calculate that the solution of the differential equation

$$y'(x) + y(x) = 0 \tag{1}$$

is $y(x) = Ce^x$, with $C \in \mathbb{R}$, then check if the function Ce^x really satisfies equation (1). So, check if it holds

$$D(Ce^x) + Ce^x = 0.$$

(Hint: it does not hold)

Matematiikan peruskurssi: formulas to the exam

Formulas related to sequences and finance

Arithmetic sequence $a_i = a_1 + (i-1)d$ and sum $S_n = n\frac{a_1+a_n}{2}$.
Geometric sequence $a_i = a_1 q^{i-1}$ and sum $S_n = a_1\frac{1-q^n}{1-q}$.
Cumulated capital after n interest periods is $K_n = \left(1+\frac{p}{100}\right)^n K_0$ and the original capital is $K_0 = \frac{K_n}{\left(1+\frac{p}{100}\right)^n}$.

$$\text{Even total payment loan: total payment} = A = N \cdot \frac{\left(1+\frac{p}{100 \cdot m}\right)^n \cdot \frac{p}{100 \cdot m}}{\left(1+\frac{p}{100 \cdot m}\right)^n - 1},$$

where N = amount of loan, p = interest, n = number of payments and m = number of payments during one interest period.

Rules of linear algebra

$$\begin{array}{cccc} (A^\top)^\top = A & (rA)^\top = rA^\top & (A+B)^\top = A^\top + B^\top & (AC)^\top = C^\top A^\top \\ AA^{-1} = A^{-1}A = I & \det A^{-1} = \frac{1}{\det A} & (A^{-1})^{-1} = A & (AB)^{-1} = B^{-1}A^{-1} \end{array}$$

Rules of differentiation

$$\begin{array}{ccc} Dx^r = rx^{r-1} & De^x = e^x & D\ln x = \frac{1}{x} \\ D\sin(x) = \cos(x) & D\cos(x) = -\sin(x) & D\tan(x) = 1+\tan(x)^2 \end{array}$$

$(fg)'(x) = f'(x)g(x) + f(x)g'(x)$
$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$
$Dg(f(x)) = (g \circ f)'(x) = (g' \circ f)(x)f'(x) = g'(f(x))f'(x)$

Rules of integration

$\int x^r \mathrm{d}x = \frac{1}{r+1}x^{r+1} + C$ kun $r \neq -1$, $\int \frac{1}{x}\mathrm{d}x = \ln|x| + C$, $\int e^x \mathrm{d}x = e^x + C$, $\int \sin(x)\mathrm{d}x = -\cos(x) + C$, $\int \cos(x)\mathrm{d}x = \sin(x) + C$
Integration by parts: $\int f'(x)g(x)\mathrm{d}x = f(x)g(x) - \int f(x)g'(x)\mathrm{d}x$
Integration by substitution:
$\int g(\underbrace{f(x)}_{=t})\underbrace{f'(x)\mathrm{d}x}_{=\mathrm{d}t} = \int g(t)\mathrm{d}t = G(f(x))$ or $\int f(\underbrace{x}_{g(t)})\underbrace{\mathrm{d}x}_{g'(t)\mathrm{d}t} = \int f(g(t))g'(t)\mathrm{d}t$.

Definite integral: $\int_a^b f(x)\mathrm{d}x = \Big/_a^b F(x) = F(b) - F(a)$

-with integration by parts $\int_a^b f'(x)g(x)\mathrm{d}x = \Big/_a^b f(x)g(x) - \int_a^b f(x)g'(x)\mathrm{d}x$

-with integration by substitution $\int_a^b f(x)\mathrm{d}x = \int_{g^{-1}(a)}^{g^{-1}(b)} f(g(t))g'(t)\mathrm{d}t$.

Solving differential equations

$y'(x) = f(x) \quad \rightarrow \quad y(x) = F(x) + C$
$y'(x) = \frac{g(x)}{h(y(x))} \quad \rightarrow \quad H(y(x)) = G(x) + C, \quad C \in \mathbb{R}$
$y'(x) + f(x)y(x) = g(x) \quad \rightarrow \quad y(x) = \left(\int e^{F(x)}g(x)\mathrm{d}x + C\right)e^{-F(x)}, \quad C \in \mathbb{R}$