QUANTUM MECHANICS I A (FYSA231), spring 2010

Exercise 5.

1. Normalized wave function of the ground state of hydrogen atom is

$$\psi(\vec{r}) = (\pi a_0^3)^{-1/2} e^{-r/a_0},$$

Calculate the most probable value for r. Comaper the result with exercise 4/4 result and 2/5 result. What do you notice?

2. Show by using Dirac's delta-function $\int_{-\infty}^{\infty} \mathrm{d}x \delta(x) f(x) = f(0)$ that following are true
a) $\int_{-\infty}^{\infty} \mathrm{d}x \delta(x-a) f(x) = f(a)$
b) $\int_{-\infty}^{\infty} \mathrm{d}x \; x\delta(x) f(x) = 0.$
c) $\delta(ax) = \frac{1}{|a|}\delta(x), \text{ when } a \neq 0.$
d) $\int_{-\infty}^{\infty} \mathrm{d}x \delta'(x) f(x) = -f'(0).$

3. Let us study the time development of a 2-state system and how to calculate the probabilities related to the measured observables. Assume that $|1\rangle$ ja $|2\rangle$ are orthonormal eigenvectors. Hamilton's operator of the system is defined as:

$$\begin{aligned} \hat{H}|1\rangle &= -E|1\rangle \\ \hat{H}|2\rangle &= E|2\rangle. \end{aligned}$$

Let there be also an observable A and corresponding operator $\hat{A}$, so that

$$\begin{aligned} \hat{A}|1\rangle &= -i\hbar|2\rangle \\ \hat{A}|2\rangle &= i\hbar|1\rangle. \end{aligned}$$

a. Energy of the system is measure at time $t = 0$ and the result is $-E$. After this energy is measured at time $t = \frac{\pi\hbar}{3E}$. What are the possible results and which probability one gets them?
b. One measures A at time ä $t = 0$ and gets $\hbar$. After this one measure energy at the time $t = \frac{\pi\hbar}{3E}$. What are the possible results and which probability one gets them?
c. One measures A at time $t = 0$ and gets result $\hbar$. After this A is measured again at time $t = \frac{\pi\hbar}{3E}$. What are the possible results and which probabilty one gets them?
d. Is A constant of the motion of the system?

4. Potential $V(x)$ is defined as

$$V(x) = \begin{cases} 0, \ if \ x \leq 0, \\ -V_0, \ if \ 0 < x < L \\ 0, \ if \ L \leq x. \end{cases}$$

Particle beam moves along x-axis. Assume that incoming wave has shape $\phi = Ae^{ikx}$.

a) Find the equations for solving the amplitudes.

b) Show that the amplitude of the reflected wave is

$$A_2 = e^{iqL} \frac{2i(q^2 - k^2)\sin(qL)}{(q+k)^2 - (q-k)^2 e^{2iqL}},$$

missä $\hbar^2 k^2/(2m) = E$ and $\hbar^2 q^2/(2m) = E + V_0$

5. Continues.

a) Show that the reflection coefficient is

$$R = \frac{(q^2 - k^2)^2 \sin^2(qL)}{4q^2k^2 + (q^2 - k^2)^2 \sin^2(qL)}.$$

b) use notations $k^2L^2 = \varepsilon$ and $(q^2 - k^2)L^2 = \varepsilon_0$, this leads to $E = (\hbar^2/(2mL^2))\varepsilon$ and $V_0 = (\hbar^2/(2mL^2))\varepsilon_0$. Show that the transmission coefficient is

$$T = \frac{4(\varepsilon_0 + \varepsilon)\varepsilon}{4(\varepsilon_0 + \varepsilon)\varepsilon + \varepsilon_0^2 \sin^2(\sqrt{\varepsilon_0 + \varepsilon})}.$$

c) What are the values of T, when $\varepsilon \to 0$ and $\varepsilon \to \infty$? Did they met your expectations?

d) Use $\varepsilon_0 = 100$ and $\varepsilon = 0 - 300$ and draw T as a function of ε.