Bayes Teoria, ratkaisut 1 kevät 2011

1. •

$$a)P(E|F) = \frac{P(EF)}{P(F)} = \frac{P(EF)}{P(EF)+P(\overline{E}F)} = \frac{0.05}{0.05+0.10} = \frac{1}{3}$$
$$b)P(E|\overline{F}) = \frac{P(E\overline{F})}{P(\overline{F})} = \frac{P(E\overline{F})}{P(E\overline{F})+P(\overline{E}\overline{F})} = \frac{0.45}{0.45+0.40} = \frac{45}{85} = \frac{9}{17}$$
$$c)P(F|E) = \frac{P(EF)}{P(E)} = \frac{P(EF)}{P(EF)+P(E\overline{F})} = \frac{0.05}{0.05+0.45} = \frac{1}{10}$$
$$d)P(\overline{F}|E) = 1 - P(F|E) = \frac{9}{1}0$$
$$e)P(E) = P(E\overline{F}) + P(EF) = 0.05 + 0.45 = 1/2,$$
$$P(\overline{E}) = 1 - P(E) = 1/2 f)P(F) = P(EF) + P(\overline{E}F) = 0.15, \quad P(\overline{F}) = 1 - P(F) = 0.85$$

h) Tapahtumat E ja F eivät ole riippumattomia koska $P(E|F) = \frac{1}{3} \neq P(E) = \frac{1}{2}$.

2.

$$a)\ P(H_p|D) = \frac{P(H_p)P(D|H_p)}{P(H_p)P(D|H_p) + P(H_a)P(D|H_a)} = \frac{0.1 \times 0.2}{0.1 \times 0.2 + 0.9} = \frac{0.02}{0.02+0.9} = \frac{1}{46}$$
$$b)P(D) = P(H_p)P(D|H_p) + P(H_a)P(D|H_a) = 0.92, \quad P(\bar{D}) = 1 - P(D) = 0.08$$

3. Olkoon $D_i = \{$ i-biologi ei havaitse lajita $\}$ Rippumattomuuden oletuksesta seuraa

$$P(D_1D_2D_3|H_p) = P(D_1|H_p)P(D_2|H_p)P(D_3|H_p) = P(D|H_p)^3 = 0.008$$
$$P(D_1D_2D_3|H_a) = P(D_1|H_a)P(D_2|H_a)P(D_3|H_a) = P(D|H_a)^3 = 1$$

$$P(H_p|D_1D_2D_3) = \frac{P(H_p)P(D|H_p)^3}{P(H_p)P(D|H_p)^3 + P(H_a)P(D|H_a)^3}$$
$$= \frac{0.1 \times (0.2)^3}{0.1 \times (0.2)^3 + 0.9} = \frac{8(10)^{-4}}{8(10)^{-4} + 0.9} = 0.0008881$$

4. Rippumattomuuden oletuksesta seuraa

$$P(D_1D_2D_3|H_p) = P(D_1|H_p)P(D_2|H_p)P(D_3|H_p) = 0.6 \times 0.5 \times 0.3 = 9/100$$
$$P(D_1D_2D_3|H_a) = P(D_1|H_a)P(D_2|H_a)P(D_3|H_a) = P(D|H_a)^3 = 1$$

$$P(H_p|D_1D_2D_3) = \frac{P(H_p)P(D_1D_2D_3|H_p)}{P(H_p)P(D_1D_2D_3|H_p) + P(H_a)} = \frac{0.1 \times 9/100}{0.1 \times 0/100 + 0.9} = 0.0099$$

5. Olkoon A tapahtuma jossa voitetaan, $x = 1000$ euroa palkinto $V = x\mathbf{1}_A$ satunnainen voitto, A tapahtuma jossa voitetaan, h =panos että peli olisi reilu

$$E(V) = xP(A) = h$$
$$P(A) = h/x = \frac{1000}{40000} = \frac{1}{40}$$

6.

$$\begin{aligned} P(T) &= 1 - P(T) = 0.992 \\ P(T|Pos) &= \frac{P(T)P(Pos|T)}{P(T)P(Pos|T) + P(S)P(Pos|S)} \\ &= \frac{0.992 \times 0.001}{0.992 \times 0.001 + 0.008 \times 0.98} = 0.1123 \end{aligned}$$

Kun $P(S) = 0.7$, $P(T) = 1 - P(S) = 0.3$

$$\begin{aligned} P(T|Pos) &= \frac{P(T)P(Pos|T)}{P(T)P(Pos|T) + P(S)P(Pos|S)} \\ &= \frac{0.3 \times 0.001}{0.3 \times 0.001 + 0.7 \times 0.98} = 0.00043713 \end{aligned}$$

7. $P(EH) > 0$

$$P(E|H) = \frac{P(EH)}{P(H)} \stackrel{?}{=} P(H|E) = \frac{P(EH)}{P(E)}$$
$$\iff P(H) = P(E)$$

8. Olkoon $V = \{$ poimittu noppa on virheellinen $\}$, X nopanheiton tulos. $P(V) = 1/4$

$$a) \quad P(V|X=3) = \frac{P(V)P(X=3|V)}{P(V)P(X=3|V) + P(\bar{V})P(X=3|\bar{V})}$$

Huomataan että $P(X = 3|V) = P(X = 3|\bar{V}) = \frac{1}{/}6$. Tästä seuraa $P(V|X = 3) = P(V) = 1/4$. Siis havainto $\{X = 3\}$ ei tuo informaatiota V:stä

$$b) \quad P(V|X=1) = \frac{P(V)P(X=1|V)}{P(V)P(X=1|V) + P(\bar{V})P(X=1|\bar{V})} =$$
$$\frac{P(V)P(X=1|V)}{P(V)P(X=1|V) + P(\bar{V})P(X=1|\bar{V})} = \frac{1/4 \times 1/3}{1/4 \times 1/3 + 3/4 \times 1/6} = 0.4$$

Kun havainto $\{X = 1\}$ tuo lisää informaatiota V:stä.

$$b) \quad P(V|X=2) = \frac{P(V)P(X=2|V)}{P(V)P(X=2|V) + P(\bar{V})P(X=2|\bar{V})} = 0$$

koska $P(X = 2|V) = 0$. Havainto $\{X = 2\}$ on maksimaalisesti informaatiivinen V:stä, havainnon jälkeen $\bar{V}$ on varma.