Demo 10,
Partial Differential Equations, 2021

1. Solve the problem
$$\begin{cases} \partial_{tt}u - \partial_{xx}u = x^2, & \text{in } \mathbb{R} \times (0, \infty), \\ u(x, 0) = x, \\ \partial_t u(x, 0) = 0. \end{cases}$$
Hint: See section 6.6 in lecture note.

2. Solve the problem
$$\begin{cases} \partial_{tt}u - \partial_{xx}u = xt & \text{in } \mathbb{R} \times (0, \infty), \\ u(x, 0) = 0, \\ \partial_t u(x, 0) = 0. \end{cases}$$

3. Let $L > 0$. Derive using separation of variables a formal series solution to
$$\begin{cases} \partial_t u - \partial_{xx}u = 0 & (0, L) \times (0, \infty), \\ u(0, t) = 0, u(L, t) = 0, \\ u(x, 0) = f(x), \end{cases}$$
where $0 \leq x \leq L$ and $t \geq 0$.

4. Let $L > 0$. Derive using separation of variables a formal series solution to the Neumann problem
$$\begin{cases} \partial_t u - \partial_{xx}u = 0 & (0, L) \times (0, \infty), \\ \partial_x(0, t) = 0, \partial_x(L, t) = 0, \\ u(x, 0) = f(x), \end{cases}$$
where $0 \leq x \leq L$ and $t \geq 0$.