

Partial differential equations 2021, computer demo 2

Solutions to the following exercises should be returned by 15.12 to the address jarkko.siltakoski@jyu.fi. The "demo" session 30.11 at 8:30 is used as an instruction session where you can get help to these exercises.

1. In Example 7.14 in the lecture note take $a = b = 1$ and $g : [0, 1] \rightarrow \mathbb{R}$, $g(y) = y(1 - y)$. Write a code that computes the partial sum with the first 10 terms and draw a picture. Hint: you may use the computer to evaluate the integral in the coefficient.
2. Consider the same equation as in Problem 1, but approximate the solution using the method described in Section 9.11 and draw a picture. Hint: modify the code of example 9.2.
3. Green's function was defined using the corrector function $y \mapsto \varphi^x(y)$. Draw graphs of φ^x when $\Omega = (0, 1) \times (0, 1)$ and $x \in \{(0.1, 0.5), (0.25, 0.5), (0.5, 0.5)\}$. Hint: modify the code of example 9.2.
4. Change the code of example 9.2 so that it works in $\Omega = (0, a) \times (0, b)$, $a, b > 0$. Test the code for example with the equation

$$\begin{cases} \Delta u(x, y) = \sin(x), & (x, y) \in \Omega := (0, 2\pi) \times (0, 1), \\ u(x, y) = 0, & (x, y) \in \partial\Omega. \end{cases}$$