DEMO 2,
Partial Differential Equations, 2021

In problems 1-4, solve $u = u(x, y)$.

1.
$$\begin{cases} 2\partial_x u + 3\partial_y u = 0, \\ u(0, y) = \sin y. \end{cases}$$

2.
$$\begin{cases} 2\partial_x u + 3\partial_y u = x, \\ u(0, y) = y. \end{cases}$$

3.
$$\begin{cases} 2\partial_x u + 3\partial_y u = xu, \\ u(0, y) = y. \end{cases}$$

4.
$$\partial_x u + x\partial_y u = 0.$$

5. Let f and g be two given continuous functions and let c be a constant. Solve the following initial value problem:

$$\begin{cases} f(y)\partial_x u + \partial_y u = cu, \\ u(x, 0) = g(x). \end{cases}$$

[Hint: the solution should be of the form $u(x, y) = e^{cy} g(x - \int_0^y f(z)\, dz)$.]

6. Suppose that $u \in C^1(\mathbb{R}^2)$ is a solution to

$$a(x, y)\partial_x u + b(x, y)\partial_y u = 0.$$

Show that for arbitrary $H \in C^1(\mathbb{R})$ also $H(u)$ is a solution.