Demo 9,
Partial Differential Equations, 2021

1. Consider a general wave equation $\partial_{tt}u = c^2\Delta u$, $c \neq 0$. Transform this into the standard form $\partial_{tt}v = \Delta v$. How does the solution given by d'Alembert's formula ($N = 1$) look for the general equation?

2. Let $F, G \in C^2(\mathbb{R})$. Show that $u(x,t) = F(x+t) + G(x-t)$ is a solution to the one dimensional wave equation.

3. Show that solutions $u \in C^2(\mathbb{R} \times (0,\infty))$ given by d'Alembert's formula with initial condition $g \in C^2(\mathbb{R})$ and $h \in C^1(\mathbb{R})$ are of above form.

4. Does the wave equation satisfy a max principle? Hint: try for example $u(x,t) = \int_{x-t}^{x+t} \sin(y)\, dy$ in a suitable domain.

5. Let $F \in C^2(\mathbb{R})$ and
$$u : \mathbb{R}^N \times (0,\infty), u(x,t) = F(\omega \cdot x - ct).$$
Show that u solves the equation $\partial_{tt}u - c^2\Delta u = 0$. Why are these called "planewaves"?