Metriset avaruudet, Excercise set 6
12.10. 2022

1. Let (X, d) be a compact metric space. Let $E \subset X$ be closed. Show that E is compact.

2. Let (X, d) be a metric space, $K \subset X$ be compact and $E \subset X$ be closed. Show that $E \cap K$ is compact.

3. Let (X, d) be a metric space and $K_1, \dots, K_n \subset X$ be compact sets. Show that $\bigcup_{j=1}^{n} K_j$ is compact.

4. Let (X, d_X) and (Y, d_Y) be metric spaces. Equip $X \times Y$ with one of the metrics d_1, d_2, d_∞. Show that $X \times Y$ is compact.

5. Let (X, d_X) and (Y, d_Y) be metric spaces and $f : X \to Y$ a homeomorphism. Show that X is compact if and only if Y is compact.

6. Let (X, d) be a metric space and $K \subset X$ be compact. Show that there exist $x, y \in K$ so that $d(x, y) = \mathrm{diam}(K)$.

7. Let $X = \{f \in C^0([0, 1]) : \ ||f||_\infty \leq 1\}$. Equip X with the metric d_∞ induced by the norm $||.||_\infty$. Then X is both closed and bounded. Show that X is not sequencially compact.

8. Go through the construction of the ternary $\frac{1}{3}$-Cantor set K and explain why K is both compact and non-empty.